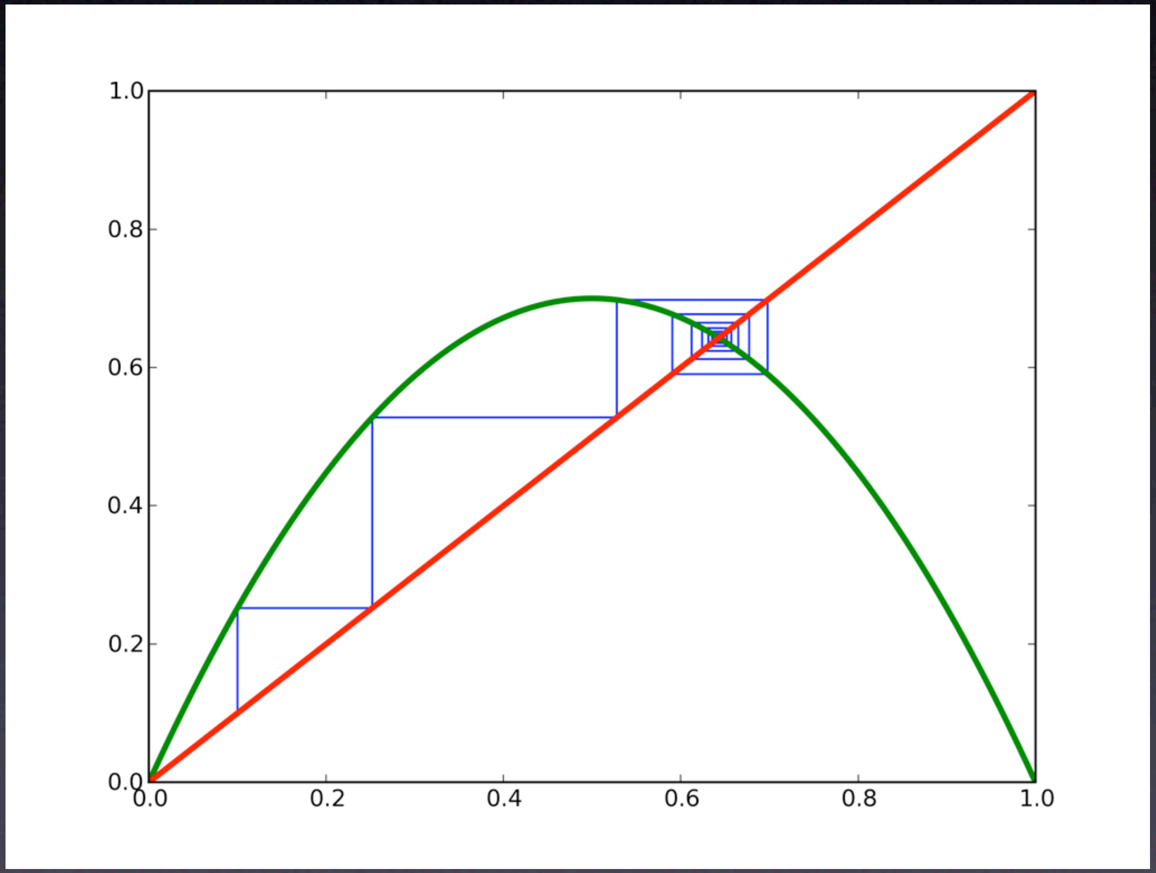
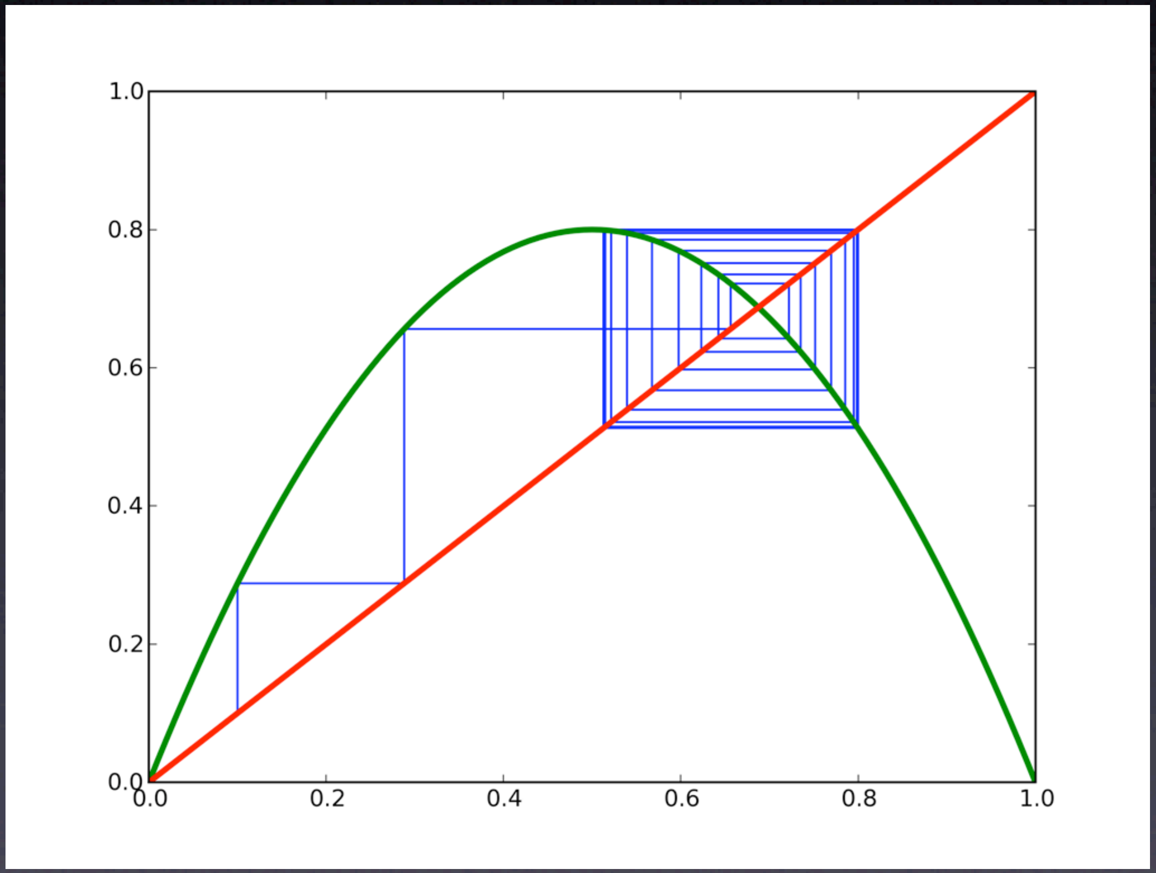
Bifurcations & Chaos in Iterated Maps I: Chaos & Lyapunov Exponents / Invariant Measure

Myers/Sethna: Computational Methods for Nonlinear Systems





Logistic map:

$$x_{n+1} = 4\mu x_n(1-x_n)$$

IterateLogistic and the args tuple

$$x_{n+1} = 4\mu x_n(1-x_n)$$

one-dimensional map (x) with one parameter (μ)

```
def Iterate(g, x0, N, args=()):
    """
    Iterate the function g N times, starting at x0
    with extra parameters passed in as a tuple args.
    Return g(g(...(g(x))...)). Used to find a point
    on the attractor starting from some arbitrary
    point x0.
    """
```

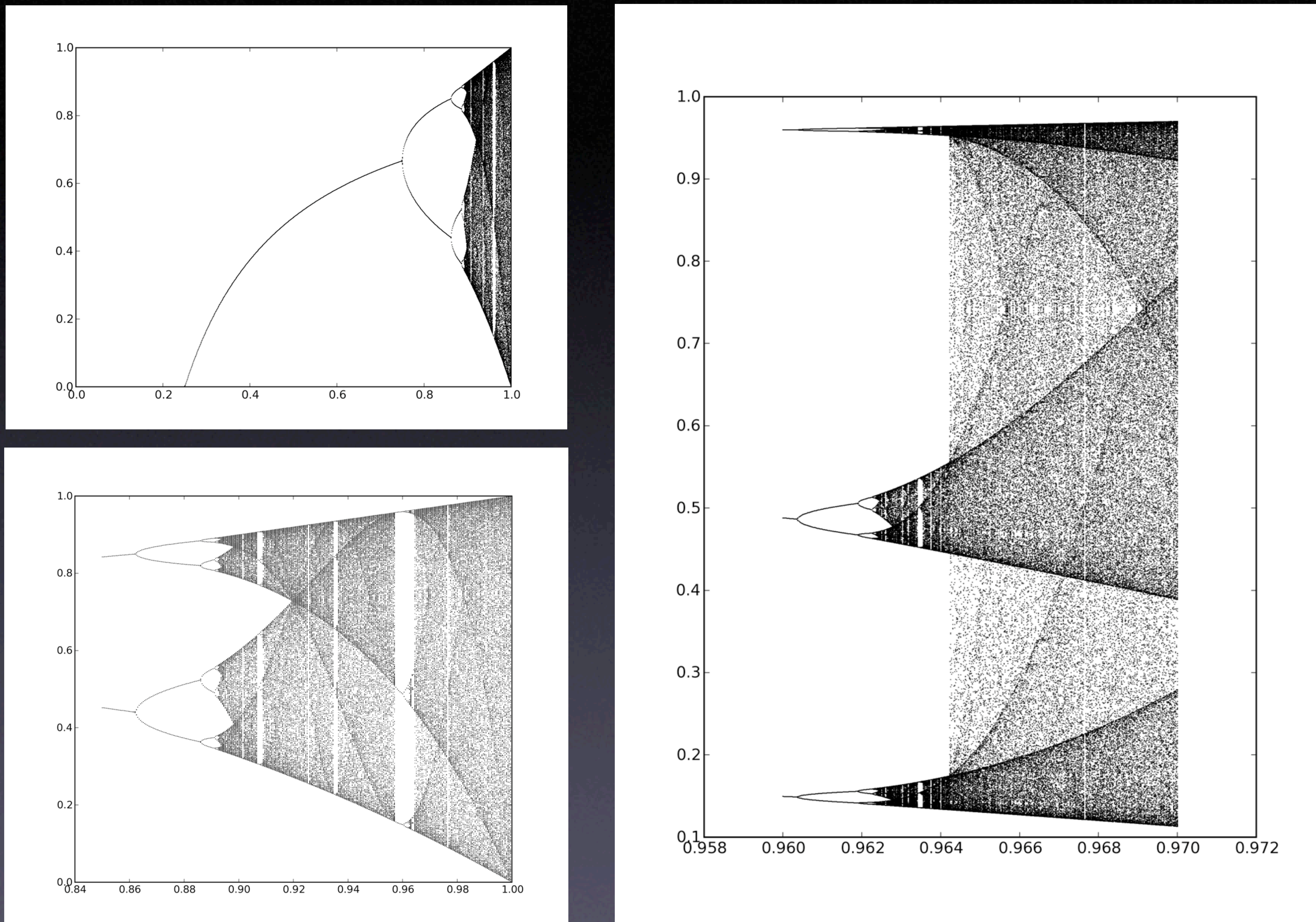
Using scipy convention of an "args tuple" to pass in an arbitrary number of extra arguments to a generic function:

e.g., scipy.integrate.odeint(dydt, y0, times, args=(), ...)
dydt is a function of y and t with optional additional arguments

dydt(y, t, a,b,c) ⇒ scipy.integrate.odeint(dydt, y0, times, args=(a,b,c))

f(x, μ): f is a function of x with additional arguments args=(μ,)

Bifurcation Diagram

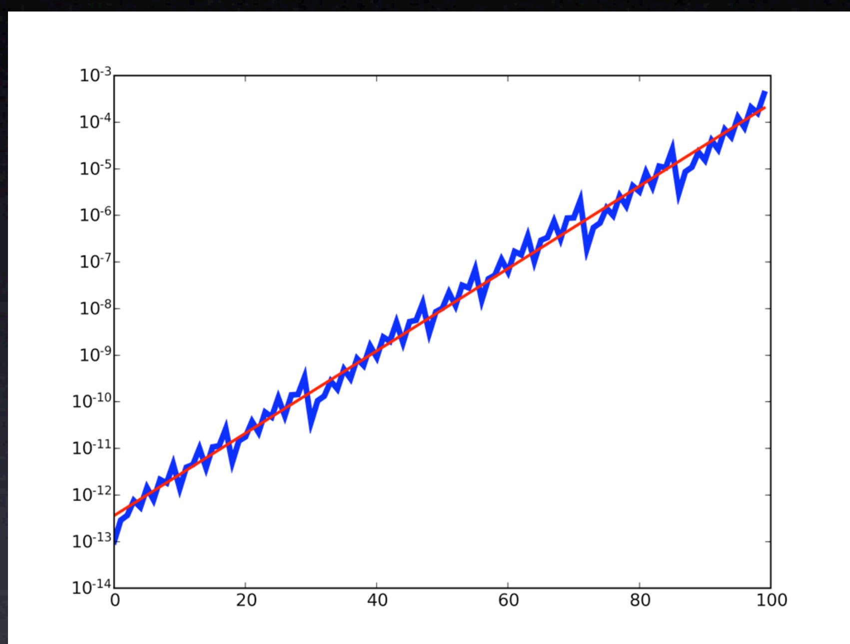


Lyapunov Exponent

rate of divergence (or convergence) of nearby trajectories

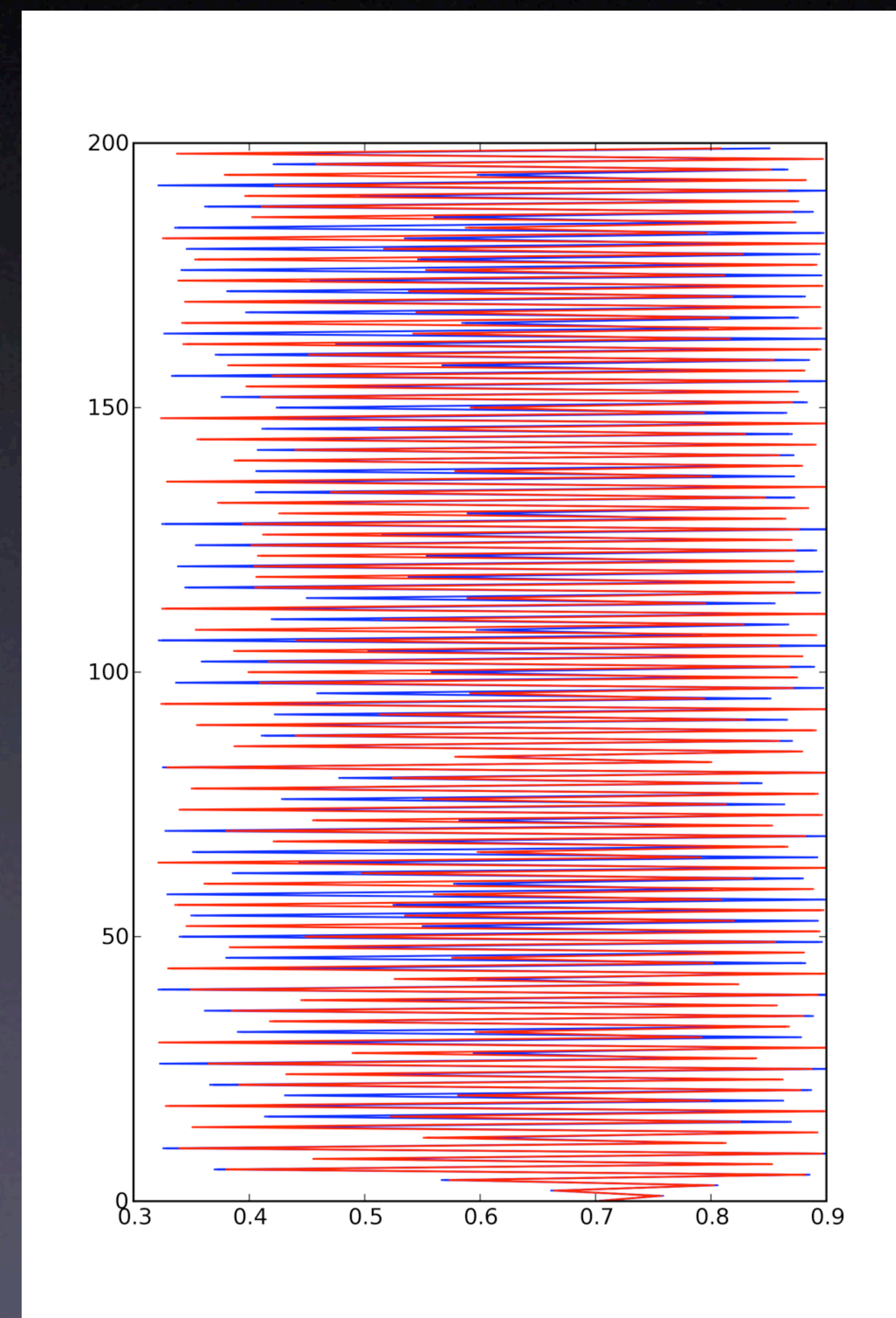
$\Delta x_{n+1} \sim \exp(\lambda t)$ → sensitive dependence on initial conditions

Δx

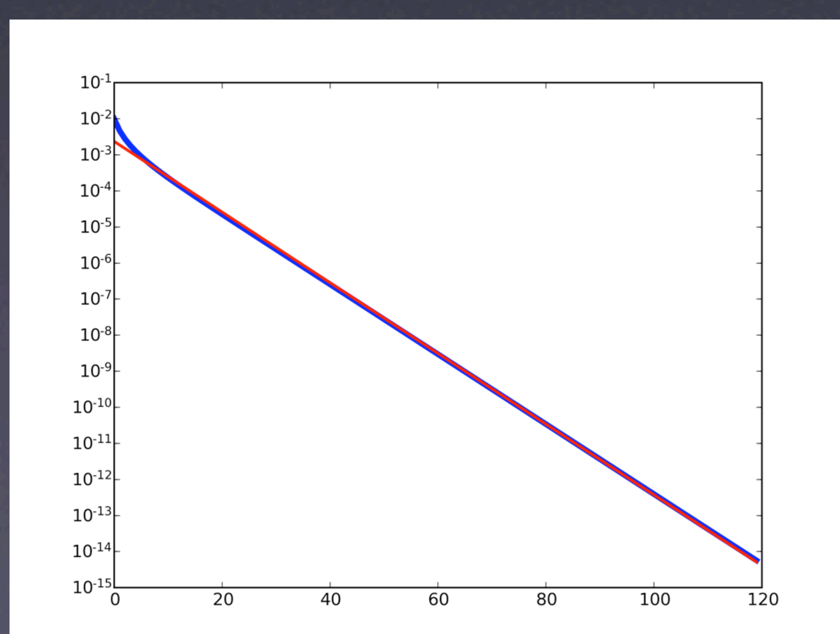


t

μ=0.9



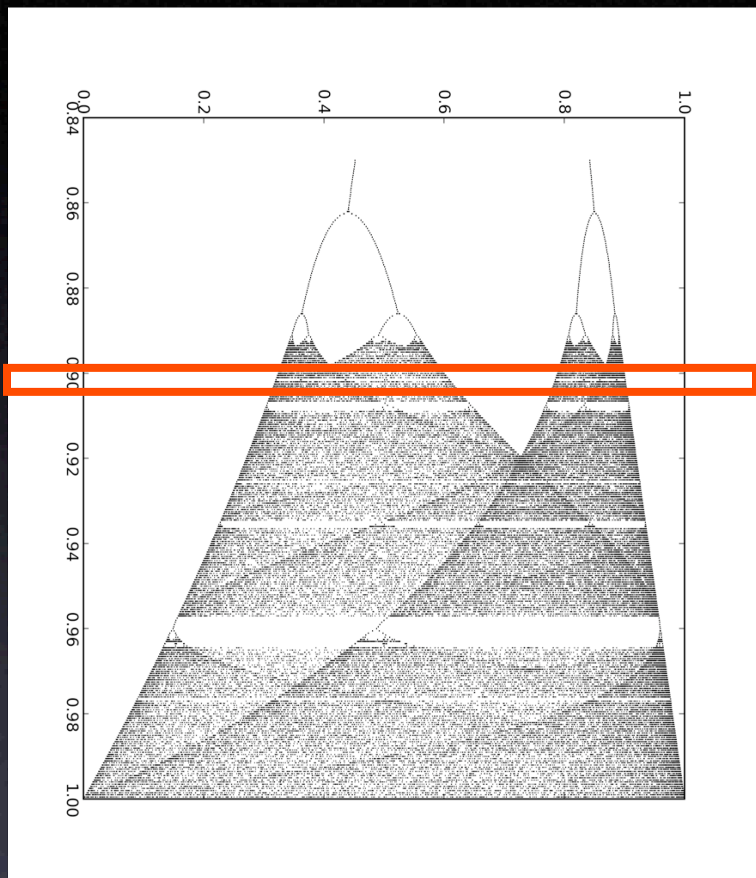
Δx

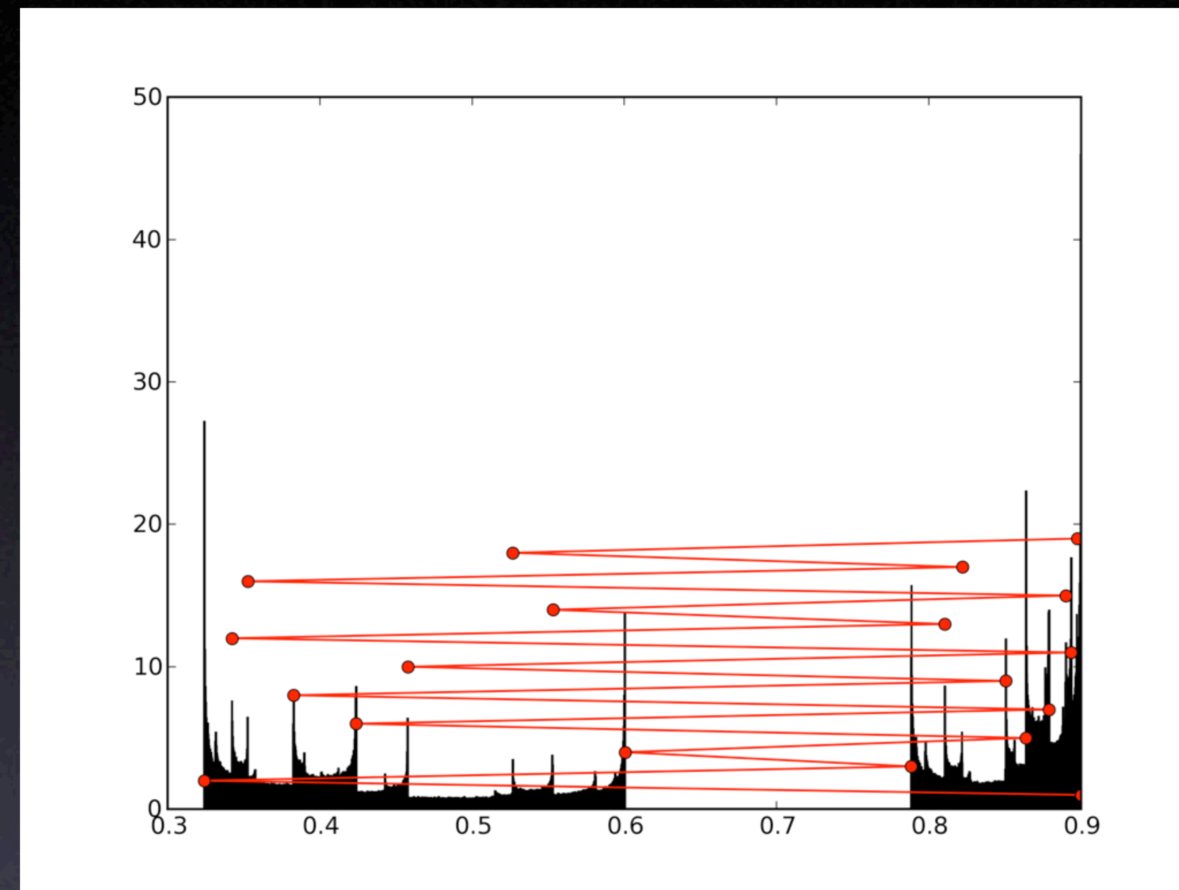


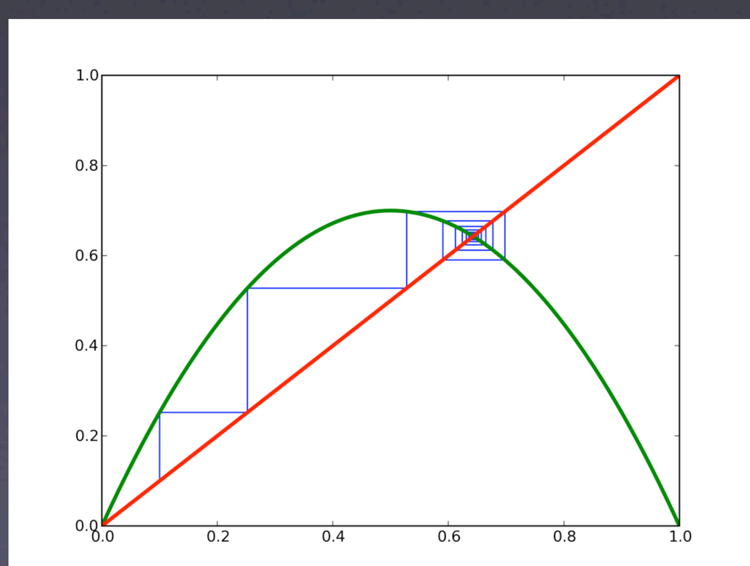
μ=0.3

Invariant Measure

stationary probability density in chaotic regime







Δy = (local slope) * Δx

- trajectories get expanded or compressed depending on value of local slope
- at the critical point of the map (at x=0.5), slope → 0 ⇒ compression → ∞

- singularities in invariant measure