Random Universality:
Random Matrix Theory and Extreme Value Statistics

Sethna / Myers Computational Methods in Nonlinear Science

Universality: ***a surprising congruence or sharing of properties or characteristics between seemingly unrelated systems, usually attributed to deep underlying truths***

• *Central Limit Theorem*: The average of several random variables always has the same (normal or Gaussian) universal distribution.
• *Extreme value statistics*: The largest of several random variables always has one of a few (Gumbel, Weibull, Fréchet) universal forms
• *Random Matrix Theory*: large matrices, wherever they come from, have eigenvalues whose spacings have one of a few universal distributions.

Gumbel Distribution

Extreme values: Biggest of Bunch

Yearly values						Five year maxima
4	144	144	100	784	→	784
256	81	324	441	196	→	441
625	361	1	841	4	→	841
25	900	100	4	576	→	900

Mississippi water level → Dike height for current administration

Maximum cost of stock → Cost of 5 year stock option

October rain level → Crop insurance cost

Probability distributions for maximum X all have Gumbel distribution

$$\rho(X) = e^{-(x+e^{-x})}$$

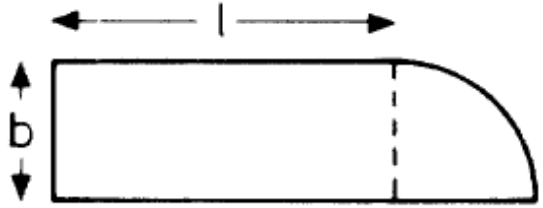
Random Matrix Theory

Eigenvalues of Large Matrices

Eigenvalue λ , ***eigenvector v*** of a matrix M

$$M \cdot v = \lambda v$$

(Directions which stretch or shrink, but don't rotate)



Irregular drum, resonant absorption

Quantum energy states of atoms, nuclei

Mechanics: moments of inertia

Waves, drum vibrations:

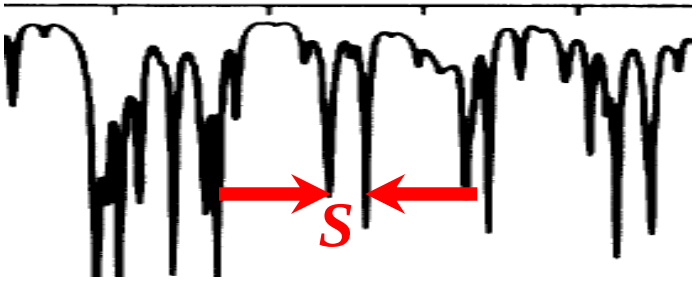
$$\partial^2 h/\partial t^2 = c^2 \nabla^2 h$$

standing wave

$$h = f(x) \cos \omega t$$

$$-\omega^2 f = c^2 \nabla^2 f = M f$$

√-Eigenvalues = resonant frequencies



Splittings S have *level repulsion*

Stöckmann and Stein, PRL 64, 2215 (1990)