Errata, first printing
Statistical Mechanics
Entropy, Order Parameters, and Complexity

James P. Sethna

Laboratory of Atomic and Solid State Physics,
Cornell University, Ithaca, NY 14853-2501

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p. 13, Exercise 1.8(c)
Change wording in the first two sentences, previously "is linear in the number of nodes ... (Hint:... for some α)", hence

> (c) *Argue that the time needed to translate the 3-colorability problem into a* **3SAT** *problem grows at most quadratically in the number of nodes M in the graph (less than αM^2 for some α for large M).* (Hint: the number of edges of a graph is at most M^2.) *Given an algorithm that guarantees a solution to ...*

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p. 32, Exercise 2.12
I got feedback from both Jean-Philippe Boucheau (a friend) and A. J. Sutter (an Amazon reviewer) that I should mention the failings of the Black–Scholes model. In reprinting we're asked not to repaginate, so the caveats are brief.... Change "In this exercise we shall ... assumptions" to

> Black and Scholes make several assumptions: no jumps in stock prices, instant trading, etc. These assumed, there is a risk-free strategy and a *fair price* for the derivative, at which no net profit is made. (The 1987 market crash may have been caused by traders using the model, a seeming conspiracy to punish those who think they can eliminate risk.) We treat a special case.

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p. 32, Exercise 2.12, second itemized entry needs to fit on two lines
Change "The stock has only two possible values at the date..." to "The stock hase one of two values at the date...", hence

- The stock has one of two values at the date of the option (the expiration date), $X_u > X_d$.[40]

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p. 44, note 25
Change
"Boltzmann's constant k_B is the"
to
"Boltzmann's constant $k_B = 1.3807 \times 10^{-23}$ J/K is the"
hence

[25]We shall see that temperature is naturally measured in units of energy. Historically we measure temperature in degrees and energy in various other units (Joules, ergs, calories, eV, foot-pounds, ...); Boltzmann's constant $k_B = 1.3807 \times 10^{-23}$ J/K is the conversion factor between units of temperature and units of energy.

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p. 109, figure 6.2
Change "i" to "j" in subscript E_j^R on left-hand side of figure (new PostScript figure provided, Fig 1)

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p. 110, equation 6.24, second line
Change "$-k_BT$" to "$+k_BT$"
p. 110, equation 6.24, third line line
Change "$-k_BT$" to "$+k_BT$"
hence

$$
\begin{aligned}
A_{\text{ideal}}^{\text{indist}} &= -k_BT \log\left((L/\lambda)^{3N}/N!\right) \\
&= -Nk_BT \log(V/\lambda^3) + k_BT \log(N!) \\
&\sim -Nk_BT \log(V/\lambda^3) + k_BT(N \log N - N) \\
&= -Nk_BT \left(\log(V/N\lambda^3) + 1\right) \\
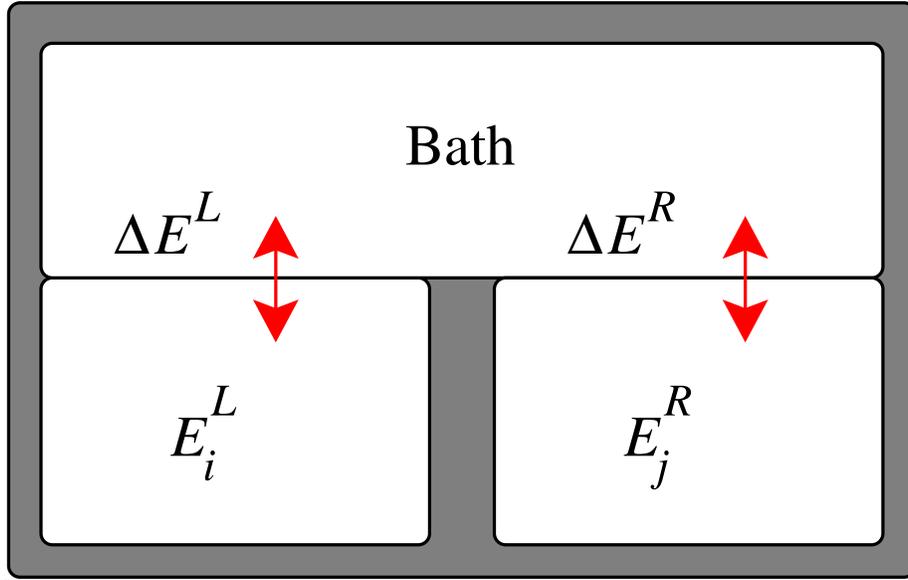&= Nk_BT \left(\log(\rho\lambda^3) - 1\right),
\end{aligned}
\tag{6.24}
$$

Figure 1: New figure 6.2

p. 146, eqn 7.56
Change "$\frac{1}{M}$" to "$\frac{1}{M!}$" in second line, first equality AND remove summation over M in second line, second equality, hence

$$
\begin{aligned}
\Xi^{\mathrm{NI,MB}} &= \sum_M \frac{1}{M!} \left(Z_M^{\mathrm{NI,MB}}\right)^M \mathrm{e}^{M\beta\mu} = \sum_M \frac{1}{M!} \left(\sum_k \mathrm{e}^{-\beta\varepsilon_k}\right)^M \mathrm{e}^{M\beta\mu} \\
&= \sum_M \frac{1}{M!} \left(\sum_k \mathrm{e}^{-\beta(\varepsilon_k-\mu)}\right)^M = \exp\left(\sum_k \mathrm{e}^{-\beta(\varepsilon_k-\mu)}\right) \\
&= \prod_k \exp\left(\mathrm{e}^{-\beta(\varepsilon_k-\mu)}\right).
\end{aligned}
\tag{7.56}
$$

p. 164, figure 8.2
Change "M" to "m" on vertical axis of plot in figure (new PostScript figure provided, Fig 2)

p. 171, note 24
Change $\mathrm{e}^{-n\tau}$ to $\mathrm{e}^{-n/\tau}$ in last inline equation, hence
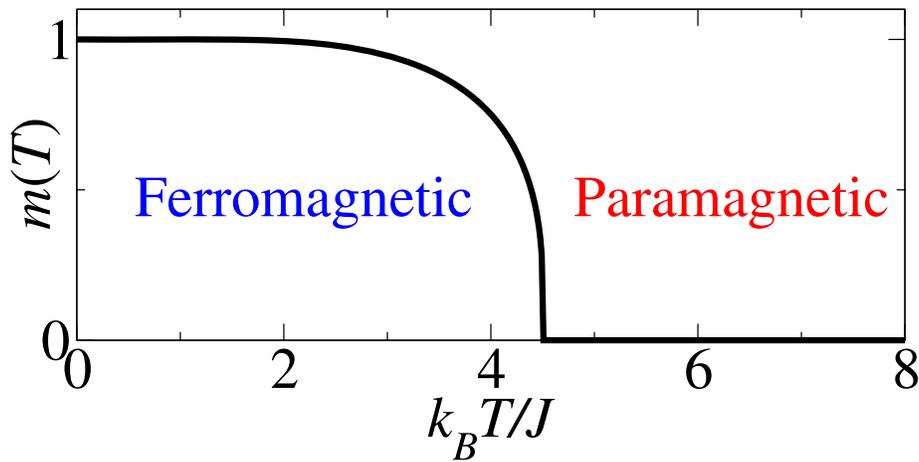
Figure 2: New figure 8.2

[24]The eigenvectors closest to one will be the slowest to decay. You can get the slowest characteristic time τ for a Markov chain by finding the largest $|\lambda_{\max}| < 1$ and setting $\lambda^n = e^{-n/\tau}$.

p. 172, figure 8.7
Change "M" to "m" on vertical axis of plot in figure (new PostScript figure provided, Fig 3)

p. 175, exercise 8.3, title
Change
"**Waiting for Godot, and Markov**"
to
"**Coin flips and Markov**"
(I was confused. The play with the coin flips was Rosencrantz & Guildenstern Are Dead, not Waiting for Godot.)

p. 217, second line:
Change "τ" to boldface "r" in inline equation, hence

...decays as a power law $C(\mathbf{r}, 0) \sim \mathbf{r}^{-(d-2+\eta)}$ at long distances...

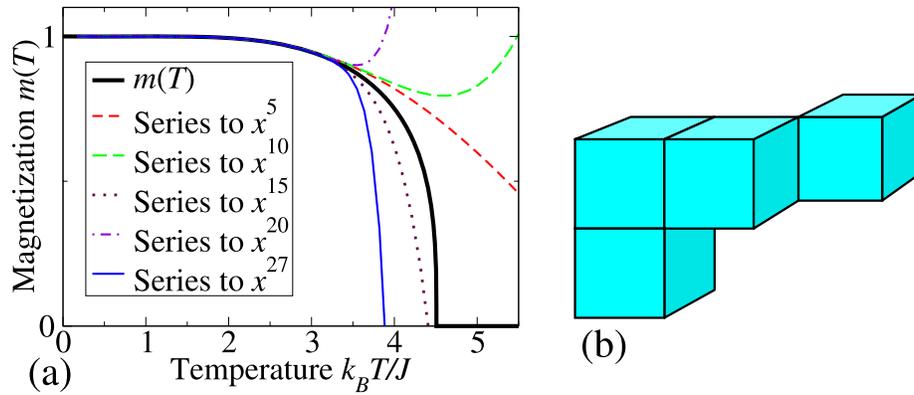p. 218, between equations 10.6 and 10.7
Change "linearize" to "expand", hence

4

Figure 3: New figure 8.7

. . . Let us assume the fluctuations are small, and expand about $\langle \rho \rangle = \rho_0$:. . .

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p. 222, second line, inline equation
Change boldface "x" to boldface "r" TWICE, hence

. . . In the case of the ideal perfume gas, the equal-time correlations (eqn 10.16) are $C^{\text{ideal}}(\mathbf{r}, 0) = 1/(\beta\alpha)\,\boldsymbol{\delta}(\mathbf{r})$, and the evolution law is given by the diffusion equation. . . .

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p. 222, equation 10.26
Change "t" to "τ" FOUR TIMES, hence

$$C^{\text{ideal}}(\mathbf{r}, \tau) = \frac{1}{\beta\alpha} G(\mathbf{r}, \tau) = \frac{1}{\beta\alpha} \left(\frac{1}{\sqrt{4\pi D\tau}} \right)^3 e^{-\mathbf{r}^2/4D\tau}. \qquad (10.26)$$

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p. 224, equation 10.37
Change $\chi(\omega)$ to $\widetilde{\chi}(\omega)$ in last term of top line, hence

$$p(\omega) = \frac{\omega|f_\omega|^2}{2} \int_{-\infty}^{\infty} \mathrm{d}\tau \ \chi(\tau) \sin(\omega\tau) = \frac{\omega|f_\omega|^2}{2} \mathrm{Im}[\widetilde{\chi}(\omega)]$$

$$= \frac{\omega|f_\omega|^2}{2} \chi''(\omega). \qquad (10.37)$$

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p. 226, equation 10.50
Change $\chi_0(\mathbf{0})$ to $\widetilde{\chi}_0(\mathbf{0})$, hence

$$\left\langle \langle s \rangle^2_{\text{space}} \right\rangle = \frac{k_B T \widetilde{\chi}_0(\mathbf{0})}{V} \qquad (10.50)$$

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p. 227, equation 10.52
Change $\mathrm{d}t'$ to $\mathrm{d}\tau$ in last integral, hence

$$s(\mathbf{x}, t) = \ldots = \int \mathrm{d}\mathbf{x}' \int_t^\infty \mathrm{d}\tau \; \chi(\mathbf{x} - \mathbf{x}', \tau) f(\mathbf{x}'), \qquad (10.52)$$

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p. 229, last line, first paragraph
Change $C(\omega)$ to $\widetilde{C}(\omega)$, hence

... fluctuations $\widetilde{C}(\omega)$; hence the name fluctuation-dissipation theorem.

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p. 230, equation 10.71
Change $\oint$ to $\int$ twice in first line, hence

$$\int_{\substack{\text{small}\\\text{semicircle}}} \frac{\widetilde{\chi}(\omega')}{\omega' - \omega}\, \mathrm{d}\omega' \approx \widetilde{\chi}(\omega) \int_{\substack{\text{small}\\\text{semicircle}}} \frac{1}{\omega' - \omega}\, \mathrm{d}\omega'$$
$$= \ldots$$
$$= \ldots. \qquad (10.71)$$

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p. 233, exercise 10.1
Equation 10.79: Change "$\widetilde{\Theta}$" to "$\widehat{\Theta}$" (tilde to widehat), hence

$$\widehat{C}(\mathbf{k}, t = 0) = |\widehat{\Theta}(\mathbf{k}, t = 0)|^2/V = A k^{n_s - 3}, \qquad (10.79)$$

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p. 233, exercise 10.1
Part (a) Change "Θ" to "$\widehat{\Theta}$" all four times it appears, hence:

(a) *Given an initial $\widehat{\Theta}(\mathbf{k}, t = 0)$ and assuming $(\partial\widehat{\Theta}/\partial t)|_{t=0} = 0$, calculate $\widehat{\Theta}(\mathbf{k}, t)$ from eqn 10.78. Calculate $\widehat{C}(\mathbf{k}, t) = \left\langle |\widehat{\Theta}(\mathbf{k}, t)|^2 \right\rangle$ in terms of A, c, and R given $n_s = 1$. For what value of L (in light years and in centimeters) will k_{220} be the first peak[34] of $k^2\widehat{C}(\mathbf{k}, t)$ at the decoupling time, if $R = 0.7$?* (Hint: $c = 3 \times 10^{10}$cm/s, $t = 380\,000$ years, and there happen to be about $\pi \times 10^7$ seconds in a year.)
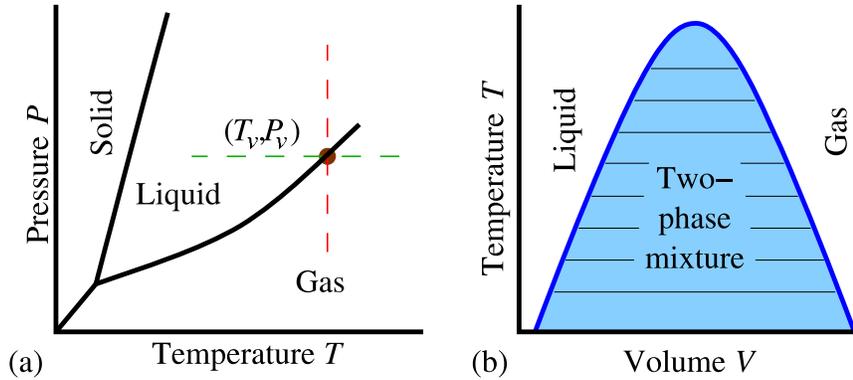
6

(a) Pressure P — Solid, Liquid, Gas — (T_v, P_v) — Temperature T

(b) Temperature T — Liquid, Two–phase mixture, Gas — Volume V

Figure 4: New figure 11.1

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p. 241, figure 11.1(a)
Change "(T_v, V_v)" to "(T_v, P_v)" label on figure (new PostScript figure provided, Fig 4)

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p. 248, second line AND equation 11.12 AND equation 11.16
Change "$\mathcal{F}_{\text{surface}}$" to "$F_{\text{surface}}$", hence

. . . domains. We will focus on the evolution of a sphere as a solvable case. The surface tension energy for a sphere of radius R is $F_{\text{surface}} = 4\pi R^2 \sigma$, so there is an inward force per unit area, (or *traction*) τ:

$$\tau = \frac{\partial F_{\text{surface}}}{\partial R} \Big/ (4\pi R^2) = 2\sigma/R. \qquad (11.12)$$

. . .

$$\Delta\mu = \frac{\mathrm{d}F_{\text{surface}}}{\mathrm{d}R} \Big/ \frac{\mathrm{d}N}{\mathrm{d}R} = (8\pi\sigma R)/(4\pi R^2 \rho) = \frac{2\sigma}{R\rho}. \qquad (11.16)$$

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p. 248, above equation 11.13
Change
"traction, with some coefficient γ:"
to
"traction and an interface mobility η:"

p. 248, equation 11.13 AND equation 11.14
Change "γ" to "η" (twice in 11.13 and three times in 11.14), hence

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\eta\tau = -\eta\frac{2\sigma}{R}. \tag{11.13}$$

We can solve for the time t_f it takes for the sphere to disappear, and hence find out how $L(t)$ grows for the non-conserved case:

$$\int_{R_0}^{0} R\,\mathrm{d}R = \int_{0}^{t_f} -2\sigma\eta\,\mathrm{d}t,$$
$$R_0^2/2 = 2\sigma\eta t_f, \tag{11.14}$$
$$L(t) \sim R_0 = \sqrt{4\sigma\eta t} \propto \sqrt{t}.$$

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p. 248 and 249
I previously confused the mobility μ with the diffusion constant D, and treated J as a volume flux rather than a number flux. This leads to several substantive changes in this section:
(a) eqns. 11.15, 11.17, 11.18, and 11.19 are altered,
(b) the sentence above equation 11.15 is altered,
(c) a portion of a sentence just below 11.15 is added,
(d) the clause after the semicolon is altered just above equation 11.16,
and (e) the sentence between equations 11.17 and 11.18 is altered ("current density" changed to "number flux per unit area").
The text in this vicinity now reads

> The argument for the case of a conserved order parameter is quite similar in spirit (Fig. 11.9). Here the curvature sets up a gradient in the chemical potential $\partial\mu/\partial x$ which causes molecules to diffuse from regions of high positive curvature to regions of low or negative curvature. The velocity of a particle will be given by the particle mobility $\gamma = D/k_B T$ (Einstein's relation) times the gradient of the chemical potential,
>
> $$v = \gamma\nabla\mu \Rightarrow J = \rho v = \rho\gamma\nabla\mu \tag{11.15}$$
>
> where J is the current per unit area and ρ is the particle density. The chemical potential change for moving a molecule from our sphere of radius R to some flat interface is just the free energy change for removing one particle; since the number of particles in our sphere is $N = \frac{4}{3}\pi R^3\rho$,
>
> $$\Delta\mu = \frac{\mathrm{d}F_{\mathrm{surface}}}{\mathrm{d}R}\bigg/\frac{\mathrm{d}N}{\mathrm{d}R} = (8\pi\sigma R)/(4\pi R^2\rho) = \frac{2\sigma}{R\rho}. \tag{11.16}$$

The distance ΔR from the surface of our sphere to another flatter surface of the same phase is (by our assumption of only one characteristic length scale) also of order R, so

$$J \sim \rho\gamma\frac{\Delta\mu}{\Delta R} \sim \frac{2\gamma\sigma}{R^2}. \tag{11.17}$$

The rate of change of volume of the droplet is number flux per unit area J times the surface area, divided by the number per unit volume ρ:

$$\frac{\mathrm{d}V_{\mathrm{droplet}}}{\mathrm{d}t} = \frac{4}{3}\pi\left(3R^2\frac{\mathrm{d}R}{\mathrm{d}t}\right)$$

$$= -\frac{A_{\mathrm{droplet}}J}{\rho} = -(4\pi R^2)\frac{2\gamma\sigma}{\rho R^2} = -\frac{8\pi\gamma\sigma}{\rho},$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{2\gamma\sigma}{\rho}\frac{1}{R^2}, \tag{11.18}$$

$$\int_{R_0}^{0}R^2\,\mathrm{d}R = \int_{0}^{t_f}-\frac{2\gamma\sigma}{\rho}\,\mathrm{d}t,$$

$$\frac{R_0^3}{3} = \frac{2\gamma\sigma}{\rho}t_f,$$

and so

$$L(t) \sim R_0 = \left(\frac{6\gamma\sigma}{\rho}t\right)^{1/3} \propto t^{1/3}. \tag{11.19}$$

This crude calculation—almost dimensional analysis—leads us to the correct conclusion that conserved order parameters should coarsen with $L(t) \sim t^{\beta}$ with $\beta = \frac{1}{3}$, if bulk diffusion dominates the transport.

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p. 252, exercise 11.2
Change "Detach the page or trace over it" to "Trace over the figure, or download a version from the text web site [126]."

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p. 275, figure caption to 12.12
Change T_1 to t in inline equation, third line from end, hence

… by a factor B and f by some factor A, so $f'(T_c - t, y) = Af(T_c - t, By) = f(T_c - Et, y)$ for large distances y.

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p. 277, second line
Change er to $(1 + e\epsilon)r$, hence

9

A system at $R = R_c + r$ after coarse-graining will be similar to a system further from the critical disorder, at $R = R_c + Er = R_c + (1 + e\epsilon)r$, so...

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p. 277, equation 12.10
Change $S^\sigma r$ to $S^{e/c}r$, hence

$$X' = S'^{e/c}r' = (S/C)^{e/c}(Er) = (S/(1 + c\epsilon))^{e/c}((1 + e\epsilon)r)$$
$$= S^{e/c}r \left(\frac{1 + e\epsilon}{(1 + c\epsilon)^{e/c}}\right) = S^{e/c}r + O(\epsilon^2) = X + O(\epsilon^2). \quad (12.10)$$

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p. 291, exercise 12.9(g)
Change "plot $T[T[f_{\sin}]]$ and compare with $T[T[f]]$" to "compare $T[T[f_{\sin}]]$ to $T[T[f]]$ at their onsets of chaos", hence

(g) Universality and the renormalization group: *Using the sine function of eqn 12.40, compare $T[T[f_{\sin}]]$ to $T[T[f]]$ at their onsets of chaos. Are they approaching the same fixed-point?*

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p. 336, sub-index entry for "Ideal gas!free energy density functional"
Change
"linearized" to
"expanded about ρ_0"

Rear endpaper, paperback version:
Weird printing problems with four figures on lower right-hand side (striped martensite, snowflake, and two Ising model square-grid figures).