**Physics 218: Waves and Thermodynamics**
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**Hints for Problem Set 2**
Latest revision: August 26, 2003, 2:00 pm

I'm supplying hints that should facilitate your doing problem set 2. You may want to work on them first without looking at the hints. No guarantees that the hints won't just be confusing: it's hard to guess what additional information might be helpful.

Your answer for problem 1.2.3 should be similar in spirit to the discussion of reflection off fixed and free boundaries in lecture. In problem 1.3.2 (b), take the real part of both sides of $\exp(i(x+y)) = \exp(ix)\exp(iy)$.

The quick ones should not need hints. The fifth natural mode has five antinodes.

Problem 2.1(b) should follow from (a) just by plugging in for the two second derivatives. In problem 2.1(c), you need to put boundary conditions not only at $x = 0$ and $x = L$, but also at $t = 0$ and $t = -\delta t$. If you use a spreadsheet to do it, the boundary conditions will look something like:

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | ... | 12.5 | 13 | 13.5 | 14 | 14.5 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | | | | | | | | | | | | | | |
| -0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ... | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ... | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.1 | 0.00016617 | | | | | | | | | | | | | 0 |
| 0.2 | 0.00022263 | | | | | | | | | | | | | 0 |
| 0.3 | 0.000296786 | | | | | | | | | | | | | 0 |
| 0.4 | 0.000393669 | | | | | | | | | | | | | 0 |
| 0.5 | 0.000519575 | | | | | | | | | | | | | 0 |
| ... | ... | | | | | | | | | | | | | ... |
| 19.7 | 4.18617E-21 | | | | | | | | | | | | | 0 |
| 19.8 | 2.10494E-21 | | | | | | | | | | | | | 0 |
| 19.9 | 1.05315E-21 | | | | | | | | | | | | | 0 |
| 20 | 5.24289E-22 | | | | | | | | | | | | | 0 |

Problem 2.2 does one of a few Fourier series that isn't too messy to do by hand: more interesting problems are usually done numerically, using FFT's (Fast Fourier Transforms). (The other Fourier problem that's done analytically is the Gaussian, which you'll see soon.) Do the integrals for the step function by breaking them into two integrals, one from $(0, L/2)$ and one from $(L/2, L)$: do them as complex exponentials. The main idea is to convince yourself that the Fourier series does converge to the original function, and to give you a hint as to why the mathematicians need to work hard to prove it. (In particular, our series is converging pointwise, but not uniformly.) The triangle function illustrates both that it's easy to do derivatives and integrals using Fourier series, and that the convergence is better when there are no jumps in the function.