Physics 218: Waves and Thermodynamics
Fall 2003, James P. Sethna
Homework 6, due Wednesday Oct. 15
Latest revision: October 4, 2003, 9:33 am

Reading

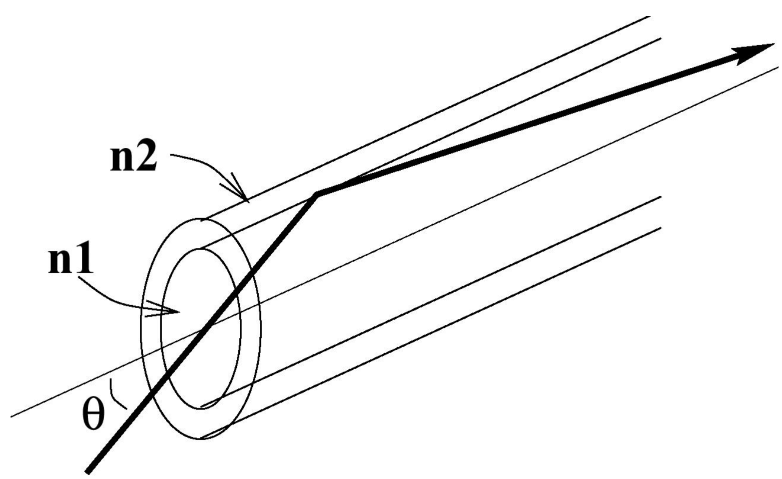
Elmore & Heald, sections 5.5-5.7 (Waves in 3D Fluids)
Feynman, section I.26 (Least Time), I.27 (Geometrical Optics), I.28 (Dipole Radiator), I.29 (Interference), II.20-4 (Spherical Waves)

Experimental Lab II

Microwaves and Optics, Monday evening 10/6 and Tuesday afternoon 10/7, Rock B26 and B30.

Problems

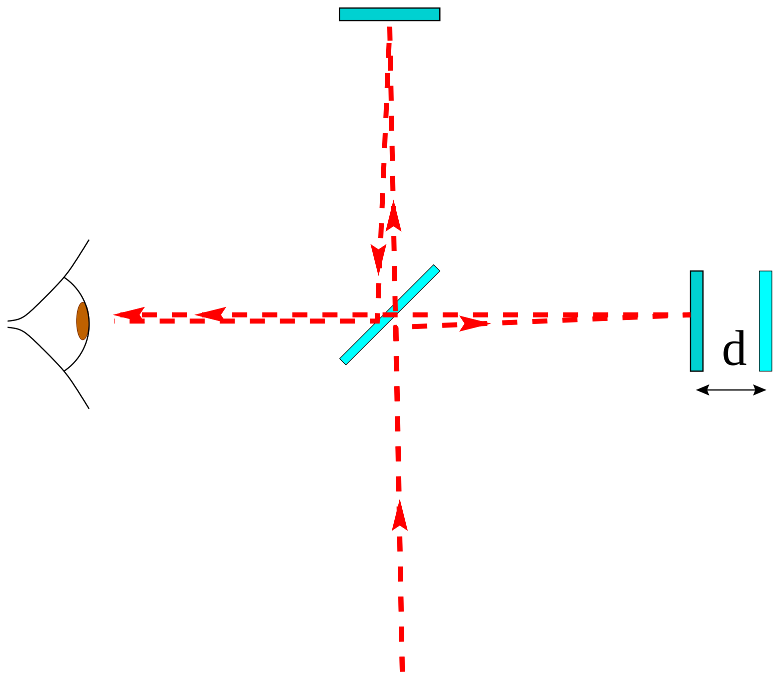
(6.1) Optical Fibers and Total Internal Reflection. An optical fiber consists of a glass core (index of refraction n_1) surrounded by a coating (index of refraction $n_2 < n_1$). Suppose a beam of light enters from air obliquely at an angle θ with the fiber axis as shown in the figure below.



(a) Show that the greatest possible value of θ for which a ray can be propagated down the fiber without leaking out is given by $\theta = \sin^{-1}(n_1^2 - n_2^2)^{1/2}$. Assuming that the glass and coating indices of refraction are 1.55 and 1.50, respectively, calculate θ_{max}.

(b) What would the critical angle be if the outer layer of glass were not there?

(6.2) Michelson Interferometer.



As one of the mirrors of a Michelson interferometer is moved through a distance d of 0.163 mm, 500 bright fringes move across the field of view. What is the wavelength of the light illuminating the mirrors of the interferometer?

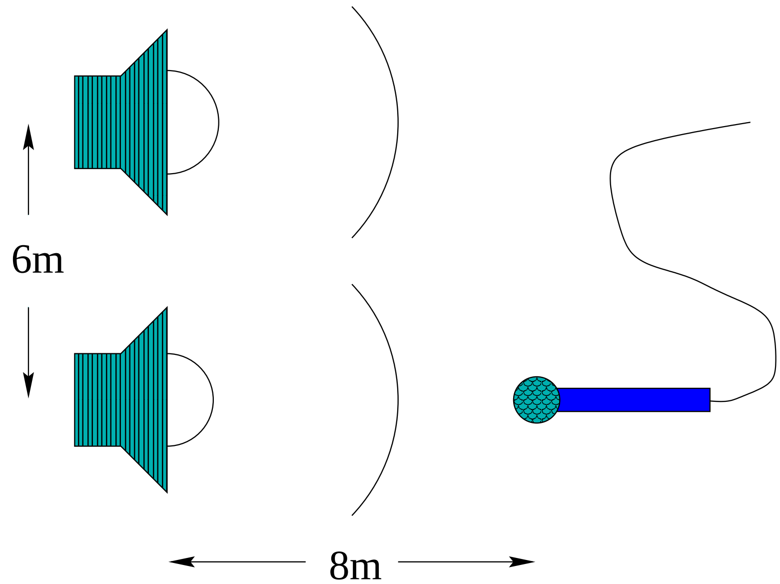
(6.3) Reflectionless Coatings. A string has three segments, the first of densities $\mu_1 =$ 0.1kg/m, the second a short segment of density $\mu_2 = 0.05$ kg/m, followed by a segment of density $\mu_3 = 0.025$. The string is under tension $\tau = 160$N. Sinusoidal waves of frequency $\omega = 300$rad/s impinge from the left.

(a) How long should the middle segment be to minimize the reflection?

This is an example of a *reflectionless coating.* Your glasses may have such a coating, designed to reduce the reflections of light from their surface. (It's much more work to design one that works at all wavelengths. . .)

(b) Check your answer to part (a) with Pythag. The REFJUMP preset should set things up properly: change X12 or X23 on the *Configure* menu to change the length of the middle, red segment. Test to make sure your answer does indeed give less reflection than longer or shorter segments. (Zooming in on the reflected pulse on the $y(t)$ plots makes it easy to measure the amplitude to high accuracy.)

(6.4) Sound Wave Interference.



Suppose there are two loudspeakers emitting spherical sound waves, a distance $d = 6m$ apart along the y axis (at $x = 0, y = \pm 3$m). The sources emit sound at the same frequency, and are in phase. Consider the point B at $x = 8$m, $y = 3$m, directly in front of one of the loudspeakers. If the wavelength of sound is two meters, is there constructive or destructive interference? How about a wavelength of 4m? Check these qualitatively using the program Huygens, which you downloaded along with Pythag for an earlier assignment. (Put "X Screen" to 8 m, d to 6 m, and the screen size to something sensible.) Is the intensity exactly zero for the case of destructive interference?* (Zoom in on the graph with the right mouse button.) Why not? What relative intensity I_{+3}/I_{-3} of the sources would produce zero sound level at B for the destructive case, for point sources of sound?

(6.5) Double Thin Slit. A double slit with slit separation d is illuminated by coherent light of wavelength λ. The lower slit is covered by a piece of glass of thickness t and refractive index $n = 1.3$. An interference pattern is observed on a screen a distance $D >> d$ away. (a) At what angle θ will the principle $m = 0$ maximum of the interference pattern be? (You may assume that θ is small.) (b) At what minimum thickness will the interference pattern show destructive interference at $\theta = 0$?

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* Huygens simulates a source which is a thin slit, rather than a point source, so the decay of amplitude with distance is different than the one for the analytical portion of this problem.