$$\boxed{\text{What Is Entropy?}}$$

Review: Statistical Mechanics Entropy
→ Thermodynamics Entropy

(Momenta for N Ideal Gas Molecules)

Universe = Bath + System $S = S_{bath} + S_{system}$

$E = E_{Bath} + E_{system}$ $\Omega = \Omega_{bath} \cdot \Omega_{system}$

Boltzmann & Temperature In General

$\cancel{\Omega_P(E_B) = (\pi^{3}) (2m(E_B))^{3N/2}}$ ← ~~Surface Area of Sphere~~ in $\vec{P}$-Space 3N dimensions momentum space

$$= \Omega_b(E-E_s) / \int \Omega_b(E-E')dE'$$

$$\rho(E_s) \simeq \frac{\Omega_P(E-E_s)}{\Omega_P(E)}$$ ← Equator ≈ whole sphere

→ particular state w/ energy E_S

$$= e^{\log\left[\frac{\Omega_P(E-E_s)}{\Omega_P(E)}\right]}$$

$$\boxed{S = k_B \log \Omega_P}$$

$$= e^{\log \Omega_P(E-E_s) - \log \Omega_P(E)}$$

$$= e^{[S(E-E_s) - S(E)]/k_B}$$

$E_S << E$ ← Bath doesn't cool down when E_S removed

$$\simeq e^{-E_s \frac{\partial S}{\partial E} / k_B}$$ General ⟸ definition $\boxed{\frac{1}{T} = \partial S / \partial E}$

$$\boxed{\rho(E_s) \propto e^{-E_s/k_B T}}$$ General Boltzmann

Connection: Thermo = Stat Mech

When energy E_B transfers from bath to system,

$\frac{1}{T} = \Delta S / \Delta E$ $\Delta S = \frac{\Delta E_B}{T} \sim \frac{Q}{T}$ ⇒ Thermodynamic Entropy from Carnot

Review: Thermodynamics of Piston Entropy → Statistical Mechanics Entropy

Isothermal Expansion of Ideal Gas

$$\Delta S = \int \frac{dQ}{T}$$ Thermodynamic Definition

$$dQ = P\,dV$$ No change in internal energy ($U = \frac{3}{2} NkT$ by equipartition) → heat in = work out

$$\Delta S = \int_{V_1}^{V_2} \frac{P\,dV}{T} = \int_{V_1}^{V_2} \left(\frac{NkT}{V}\right)\frac{dV}{T}$$ $P = \frac{NkT}{V}$

$$= \int_{V_1}^{V_2} Nk \frac{dV}{V} = Nk \log_e\left(V_2/V_1\right)$$

No $N!$?

Indistinguishable?

& Usually associated with ~~momentum~~ position-space volume $\Omega_{\mathbb{X}}$.

Define $S = \cancel{N k_B \ln V} = k_B \ln V^N / N!$

$$= k_B \ln L^{3N} / N! = k_B \ln \Omega_{\mathbb{X}}$$

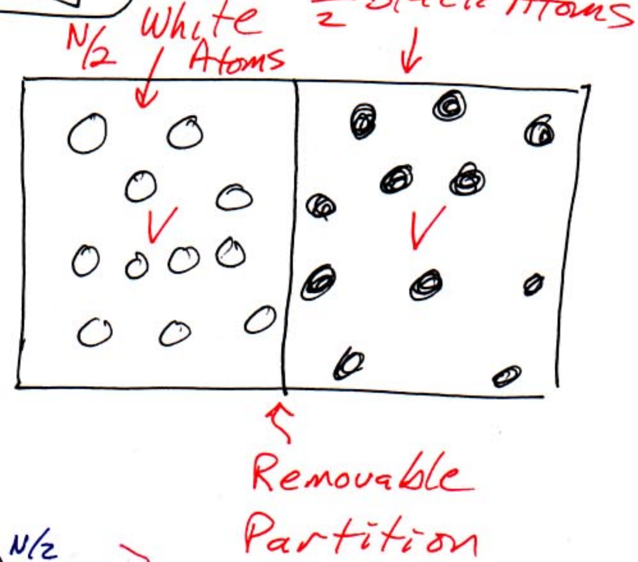
Ω = Volume in $\mathbb{X}$-space $= (\vec{X}^{(1)}, \vec{X}^{(2)}, \vec{X}^{(3)}, \ldots, \vec{X}^{(N)})$

<u>Configuration Space</u>

$$\Delta S = k_B \ln\left(V_2^N / N!\right) - k_B \ln\left(V_1^N / N!\right)$$

$$= k_B \ln\left(\frac{V_2^N / N!}{V_1^N / N!}\right) = N k_B \ln\left(V_2 / V_1\right) \checkmark$$

"Counting" Entropy: Entropy of Mixing



Separated Entropy

$$S_0 = \underbrace{k \log V^{N/2}/(N/2)!}_{\text{White}} + \underbrace{k \log V^{N/2}/(N/2)!}_{\text{Black}}$$

Mixed Entropy

$$S_1 = \underbrace{k \log (2V)^{N/2}/(N/2)!}_{\text{White}} + \underbrace{k \log (2V)^{N/2}/(N/2)!}_{\text{Black}}$$

$$= 2k \log\left[\frac{(2V)^{N/2}/\cancel{(N/2)!}}{V^{N/2}/\cancel{(N/2)!}}\right] = 2k \log 2^{N/2} = N k_B \log 2$$

$$S_1 - S_0 = \Delta S = N k_B \log 2$$

$$\boxed{\Delta S = k_B \log\left(\text{Number of } \underline{\text{choices}} \text{ of arrangements of atoms}\right)}$$

Homework: Entropy of Glasses
$= k_B \log(\#$ of different energy minima for glass atoms$)$

Quantum Mechanics:

Quantized Energies
→ All Entropy is Counting

Uncertainty Principle
→ $\Delta x \Delta p_x \sim h$

→ Volume in 6N-dimensional <u>Phase Space</u>
$\vec{\mathbb{X}} \times \vec{\mathbb{P}} = (\vec{x}^{(1)}, \vec{x}^{(2)}, \ldots, \vec{x}^{(N)}, \vec{p}^{(1)}, \vec{p}^{(2)}, \ldots, \vec{p}^{(N)})$ measured in h^{3N}

$$S = k_B \log\left(\Omega_{\mathbb{X}\times\vec{\mathbb{P}}} / h^{3N}\right)$$

Nernst's "Theorem" (Third Law):

At $T=0$, $S=0$

(Violated by Glasses? Not in Equilibrium!)

$$S = \text{Entropy} = k_B \log_e \left[\begin{array}{c} \#\text{ of "different"} \\ \text{configurations of} \\ \text{system consistent} \\ \text{with known state} \end{array} \right]$$

- Measures <u>disorder</u>: lack of knowledge about detailed arrangement of microscopic details of system

- Always increases: reducing disorder in one subsystem always demands increase in disorder in fuel used by measurer.

~~Ira Wasserman (Wednesday): Heat Death of Universe?~~

~~What Happens when Low Entropy Reservoirs Run Out?~~