Index of Refraction

Light in glass travels more slowly.

Definition: The index of refraction of a material is c/v, where c is the speed of light in vacuum and v is the speed inside the material.

Exercise: Light of wavelength 600 nm = λ impinges on a piece of glass. The speed of light in the glass is $\frac{3}{4}c$.

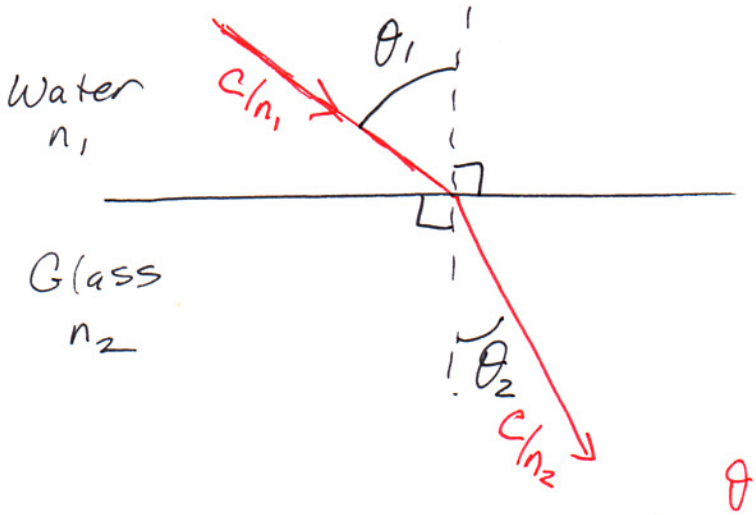
(a) What is n for our glass? $n = 1.33$
Can $n < 1$? Not without dispersion.

(b) What is c, in cm/s? v? 3×10^{10} cm/s, 2.25×10^{10} cm/s.

(c) What is λ' inside the glass? $\lambda' f = \frac{3}{4}c = \frac{3}{4}\lambda f$
What is k'/k? $\lambda' = \frac{3}{4}\lambda = 450$ nm
$k' = 2\pi/\lambda' = \frac{4}{3}k$

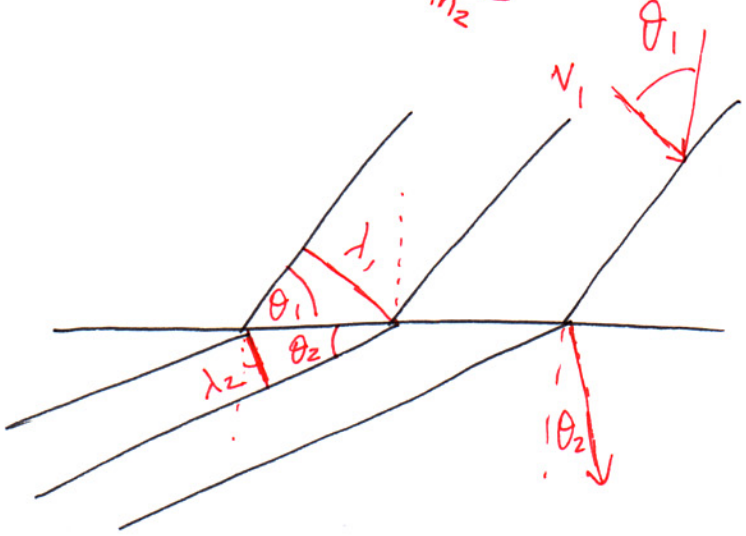
(d) What color is the light inside the glass?
Presumably, the same color as it was outside: it has to leave the glass before it can enter your eye...

Snell's Law

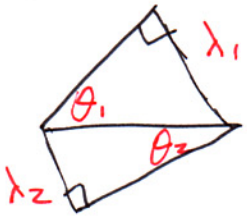


Light bends [refracts] as it goes from water to glass.

Measure θ from <u>normal</u> to surface.



Wave phase fronts must match on surface

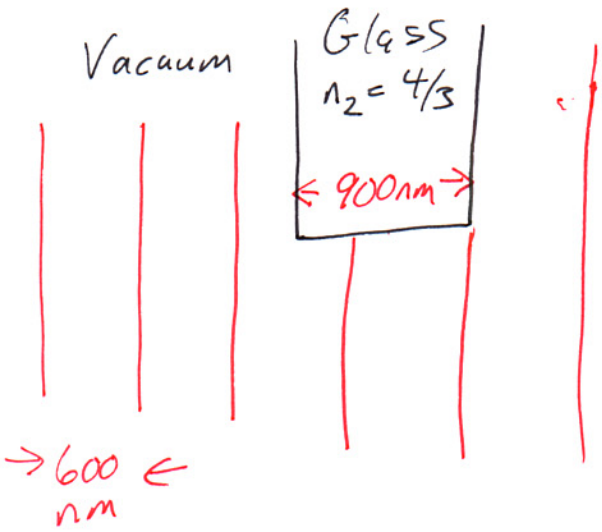


Hypotenuse $= \lambda_1/\sin\theta_1 = \lambda_2/\sin\theta_2$

$\lambda_1 = \lambda^{vac}/n_1$, $\lambda_2 = \lambda^{vac}/n_2$

$\rightarrow$ $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ Snell's Law

Thin Films



Light 600 nm hits edge of glass $n = 4/3$, 900 nm thick.

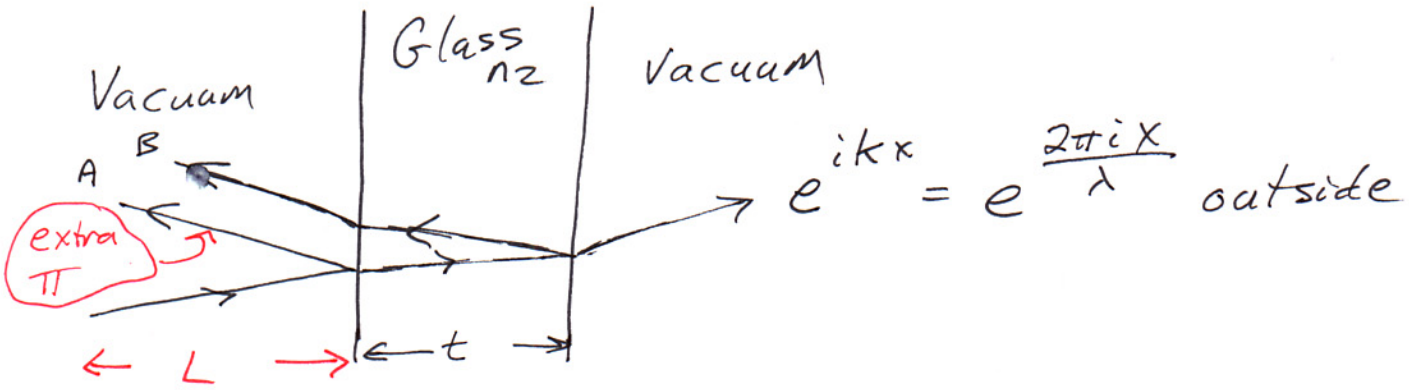
Exercise! What is the phase difference between the light passing through the glass and that passing around the glass?

$$\phi_{glass} - \phi_{vacuum} = \pi$$

($\frac{1}{2}$ wiggle)

Destructive Interference

Phase Shifts on Reflection



Which of these reflections suffers a phase shift of π, inverting the wave?

Glass has a lower velocity $\Rightarrow$ "heavy" string
Path A has an extra π phase shift.

What is the net phase difference between paths A and B? Assume normal incidence.

$$\phi_A = k(2L) + \pi$$

$$\phi_B = k(2L) + k'(2t) = k(2L) + nk(2t)$$

$$\phi_A - \phi_B = \pi - 2nkt$$

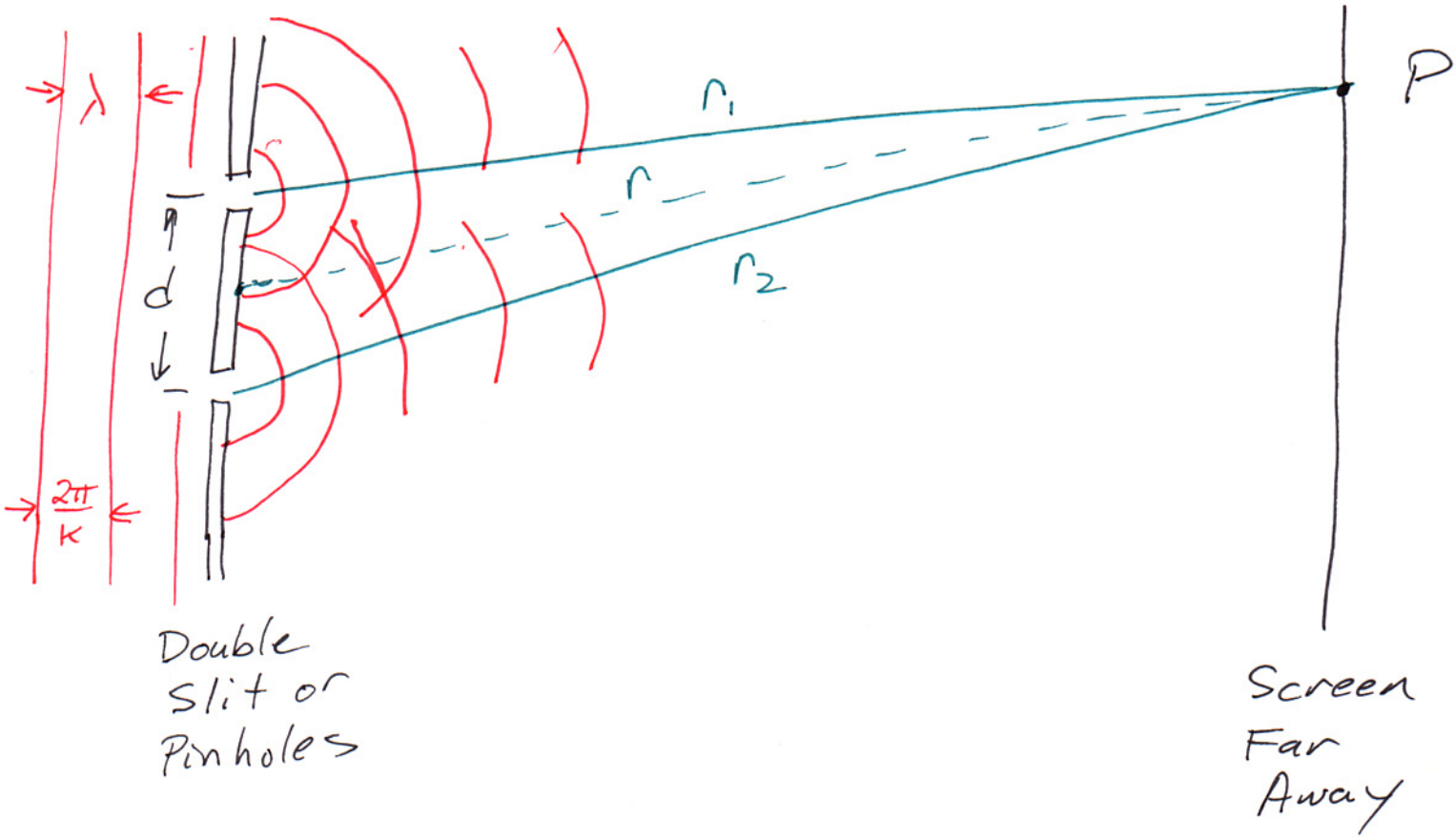
What thickness minimizes reflections? ~~Reflections~~ "string"?

$t = 0$ (obvious!)

$t = \pi/nk = \pi/k' = \lambda'/2$ (Perhaps obvious?)

Interference between Two Thin Slits

DEMO: Laser & Double Slit



Two Pinholes; Scalar Wave Equation (ignore $\vec{E}$)

$$E_p(t) = \frac{A}{r_1} e^{i(kr_1 - \omega t)} + \frac{A}{r_2} e^{i(kr_2 - \omega t)}$$

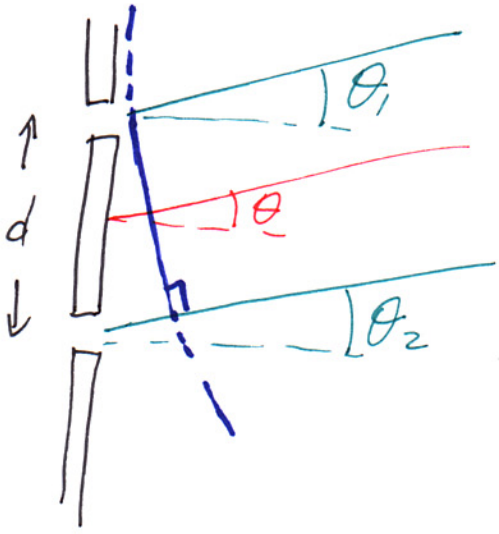
$$\approx \frac{A}{r}\left(e^{i(kr_1 - \omega t)} + e^{i(kr_2 - \omega t)}\right)$$

$$\approx \frac{A}{r} e^{i(k\bar{r} - \omega t)}\left(e^{i\phi/2} + e^{-i\phi/2}\right)$$

ϕ = phase difference

$$= k(r_2 - r_1)$$

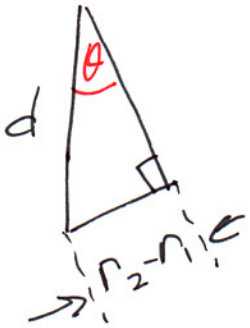
Calculating $r_2 - r_1$



For very distant points P,

- Arc of circle (blue) = straight line
- $\theta_1 \sim \theta_2 \sim \theta$

Exercise! What's $r_2 - r_1$? ϕ?



A) $d \sin\theta$

$\phi = k d \sin\theta$

$= \frac{2\pi}{\lambda} d \sin\theta$

$E_p(t) = \frac{A}{r} e^{i\omega t} \left(e^{ik \cdot r_1} + e^{i(k \cdot r_1 + \phi)} \right)$

ϕ = phase difference $= k(r_2 - r_1) = k d \sin\theta$

$= \frac{2\pi}{\lambda} d \sin\theta$

average of $\cos^2$ ⇔ ignore imaginary part

Intensity $I_{av} = \frac{A^2}{r^2} \left(\frac{1}{2}\right) \left(e^{i\frac{\phi}{2}} + e^{-i\frac{\phi}{2}} \right)^2$

$= \frac{A^2}{r^2} 2 \cos\left(\frac{\phi}{2}\right) = \frac{A^2}{r^2} 2 \cos\left(\frac{\pi d \sin\theta}{\lambda}\right)$

$= I_0$

$$E_p(t) = \frac{A}{r} e^{i(kr-\omega t)} \, 2\cos(\phi/2)$$

$$= 2E_0 \cos(\phi/2)$$

Single slit at $y=0$

Intensity

$$I_{av} = \frac{1}{T}\int \mathrm{Re}(E_p(t))^2 \, dt$$

$$= 4\frac{A^2}{r^2}\cos^2(\phi/2)\left[\frac{1}{2}\right]$$

↑ Average of cosine

$$= 4\cos^2(\phi/2)\, I_0$$

I_0: Intensity from one slit

$$= 4\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) I_0 = 4I_0\cos^2\left(\frac{kd\sin\theta}{2}\right)$$

- At $\theta = 0$ constructive interference

  $I = 4I_0$

- At ~~d~~ $d\sin\theta = \lambda/2, \; 3\lambda/2, \ldots$ destructive interference

  $d\sin\theta = \lambda, 2\lambda, -\lambda, \ldots$ constructive.

- On average, $\langle I \rangle = 2I_0$ (makes sense: two slits).