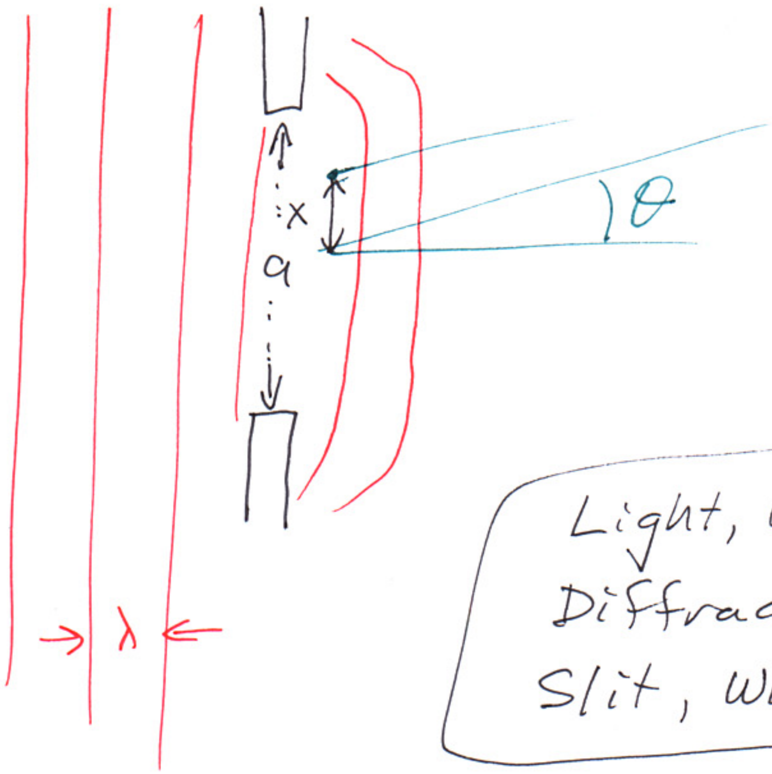
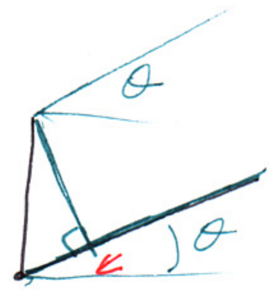
Single Slit Diffraction

Demo: Single Slit Diffraction



Light, wavelength λ
Diffracts through
Slit, width a

$$E(\theta) = \int_{-a/2}^{a/2} A\cos(\omega t + \phi(x))\,dx$$



$$= \int_{-a/2}^{a/2} A\cos(\omega t + kx\sin\theta)\,dx$$

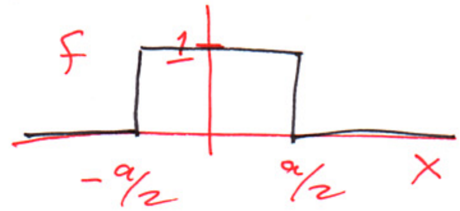
$$\phi(x) = 2\pi\frac{x\sin\theta}{\lambda} = kx\sin\theta$$

$$= \mathrm{Re}\left[\int_{-a/2}^{a/2} A e^{i(\omega t + kx\sin\theta)}\,dx\right]$$

$$= \mathrm{Re}\left[A e^{i\omega t}\int_{-a/2}^{a/2} e^{i(kx\sin\theta)}\,dx\right]$$

Let's define a slit-opening function

$$f(x) = \begin{cases} 1 & -a/2 < x < a/2 \\ 0 & x > a/2,\ x < -a/2 \end{cases}$$



Then $\tilde{f}(\check{k}) = \int_{-\infty}^{\infty} e^{i\check{k}x} f(x)\, dx$

$$= \int_{-a/2}^{a/2} e^{i\check{k}x}\, dx$$ (for our slit function)

If we pick $\check{k} = k\sin\theta$, this is in $E(\theta)$:

$$E(\theta) = Re\left(A e^{i\omega t}\, \tilde{f}(k\sin\theta)\right)$$

$$I_{av}(\theta) \propto \left|\tilde{f}(k\sin\theta)\right|^2$$

The electric field is proportional to the Fourier transform of the slit function!

For our case, $\tilde{f}(\check{k}) = \dfrac{e^{i\check{k}\frac{a}{2}} - e^{-i\check{k}\frac{a}{2}}}{i\check{k}} = \dfrac{2\sin\frac{\check{k}a}{2}}{\check{k}}$

$$E(\theta) = \left[A\cos(\omega t)\right] \frac{2\sin\left(\frac{ak\sin\theta}{2}\right)}{k\sin\theta}$$

$$\alpha = \frac{ak\sin\theta}{2} = \frac{\pi a\sin\theta}{\lambda}$$

$$E(\theta) \sim \frac{\sin\alpha}{\alpha}$$

$$I(\theta)/I_{center} = \frac{\sin^2\alpha}{\alpha^2}$$