218 Part 3: Thermodynamics

Random Behavior of Many Atoms → Simple Laws

How to connect Heat, Temperature, Entropy and Statistics to Waves?

I. Finding New Laws
- Conserved order parameter
- No time reversal symmetry

→ Diffusion Equation

"Standing" & "Travelling" Wave" Solutions

II. Derive Diffusion Equation Microscopically

~~Free Body Diagram~~ →

Random Walks

Probability Distributions

Diffusion in Gases

III. Kinetic Theory of Gases

IV. Statistical Mechanics

V. Laws of Thermodynamics

...

The Diffusion Equation

Drop of Milk in Coffee - (No Stirring)
- Gradually spreads out through whole cup
- Stirring <u>much</u> faster than waiting
- Convection, currents stir

Perfume in Air [same]

Heat diffusion through metals (fast), styrofoam (slow), earth's crust (slow)

What symmetries do these problems have?

- Homogeneous in Space $\vec{x} \to \vec{x} + \vec{r}$
- Homogeneous in Time $t \to t + \Delta$
- Isotropy under Rotations $\vec{x} \to R\vec{x}$
- Reflection in Space $x \to -x$
- ~~Reflection in Time $t \to -t$~~ $\frac{\partial}{\partial t}$ <u>allowed</u>.

What else is special?

Each of these problems describe the evolution of a <u>conserved</u> quantity

$$\rho(x,t) = \begin{cases} \cancel{\text{Coffee}}^{\text{cream}} \text{ density} \\ \text{Perfume density} \\ \text{Energy density} \end{cases}, \quad \int d^3x \, \rho(x,t) = \text{Constant}$$

[Conservation of Coffee, Energy, ...]

Finding New Laws
The Diffusion Equation

① Pick an Order Parameter Field

$\rho(x,t)$ [Energy density, coffee density, ...]

② Restrict to long time scales

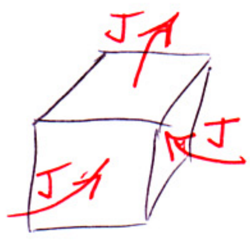
$\partial\rho/\partial t \gg \partial^2\rho/\partial t^2, \ldots$ Minimize # of time derivatives.

$$\frac{\partial\rho}{\partial t} = \mathcal{F}(\rho, \vec{x}, t, \partial\rho/\partial x, \partial\rho/\partial y, \ldots)$$

③ Impose the conservation laws.

ρ is conserved

- Can't be created or destroyed
- Can slosh from one place to another
- Call J the current of ρ



$$\int \frac{\partial\rho}{\partial t}\, dV = -\int \vec{J}\cdot d\vec{S} = \int -(\nabla\cdot J)\, dV$$

Box V
Surface $\vec{S}$

$$\boxed{\frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{J}}$$

(4) Write the most general function for J allowed by translational symmetry

$$\vec{J}_i(\rho, \cancel{\vec{x}}, \cancel{t}, \partial_j \rho, \partial_j \partial_k \rho, \ldots)$$

- Homogeneous in Space & Time
- Scalars $\rho, \partial_j \partial_j \rho, (\partial_j \rho)(\partial_j \rho), \ldots$

$$J_i = -D_{ij}(\rho, \nabla^2\rho, (\vec{\nabla}\rho)^2, \ldots)\, \partial_j \rho + E_{ijk}(\rho, \ldots)\, \partial_j \partial_k \rho + \ldots$$

$D > 0 \rightarrow J$ flows from high ρ to low ρ

(5) Restrict to long length scales

$$\rho \gg \nabla^2 \rho, (\nabla\rho)^2, \ldots$$
$$\partial_j \partial_k \rho \ll \partial_j \rho$$

$$J_i = -D_{ij}(\rho)\, \partial_j \rho$$

(6) Impose Isotropy

$D_{ij}(\rho)$ invariant under rotations

$$\Rightarrow D_{ij}(\rho) = D(\rho)\, \delta_{ij}$$

(like dielectric tensor, moment of inertia, ...)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot J = -\partial_i J_i$$

$$J_i = -D_i(\rho)\, \partial_i \rho$$

$$\frac{\partial \rho}{\partial t} = \partial_i \left(D(\rho)\, \partial_i \rho \right) = \nabla \cdot \left(D(\rho) \nabla \rho \right)$$

7 (Sometimes) Assume ρ is small, or nearly constant $D(\rho) \approx D$

$$\boxed{\frac{\partial \rho}{\partial t} = \nabla \cdot (D \nabla \rho) = D \nabla^2 \rho}$$

The Diffusion Equation