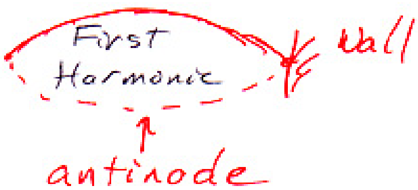
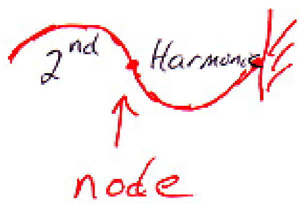
RESONANCE

Demo: Spring. Oscillate, small amplitude, increasing from low frequency. Note fundamental = first harmonic, second harmonic, ...

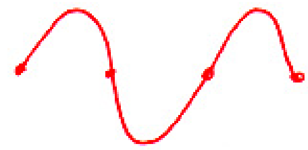
Fundamental



First overtone



Q: How many antinodes?



Demo: Sonometer. 1st Overtone = octave.

- English "resonant" = echoing
- Special forcing frequencies
  (where spring would wiggle by itself)
- Fixed boundary conditions (here)

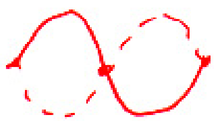
Resonant waves also called "Standing Waves" because they wiggle in place (don't travel).

They are also called "normal modes" or "eigenmodes", for reasons which will be covered later.

STANDING WAVES

For homogeneous, time-independent linear systems [future lecture], the standing waves are sines and cosines: $\sin(kx)\sin(\omega t)$

Exercise: Fill in all but the last two columns

	Antinodes	Wavelength	Wave # k	Angular frequency ω radians/s	Frequency cycles/sec
	1	$2L$	$\pi/L \left(\frac{2\pi}{2L}\right)$	$\frac{\pi c}{L}$	$f_0 = \frac{c}{L}$
	2	L	$2\pi/L$	$\frac{2\pi c}{L}$	$2 f_0$
	n	$2L/n$	$n\pi/L$	$\frac{n\pi c}{L}$	$n f_0$

Exercise: Show $y(x,t) = \sin(kx)\sin(\omega t)$ satisfies the wave equation. What is $\omega(k)$?

$$\frac{\partial y}{\partial x} = k \cos kx \sin \omega t \qquad \frac{\partial y}{\partial t} = \omega \sin kx \cos \omega t$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 \sin kx \sin \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 \sin kx \sin \omega t$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{\omega^2}{k^2}\frac{\partial^2 y}{\partial x^2}, \quad c = \omega/k, \quad \omega(k) = ck}$$

Exercise: Fill in the last 2 columns. Fixed BC → $n f_0$ all multiples of fundamental

Octave = $2 f_0$