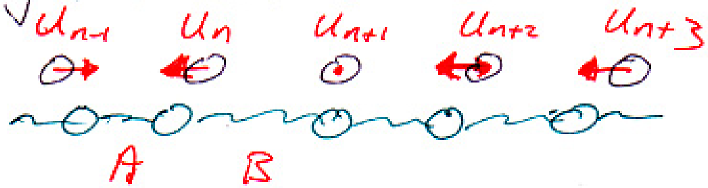
Sound and Atoms: The One Dimensional Crystal

Atom Mass M Chemical Bond = Spring Constant K

Chain of Balls and Springs,
Longitudinal Wave



δx = Spring Length
$n\,\delta x$ = Undeformed Position

$X_n = n\,\delta x + u_n$ = Current position

Bond B is stretched: length is $\delta x + u_{n+1} - u_n$
force on atom n is $K(u_{n+1} - u_n)$ to right

Bond A is squeezed: length is $\delta x + u_n - u_{n-1}$
force on atom n is $-K(u_n - u_{n-1})$ to right

note

{ If $u_n - u_{n-1} < 0$, Force on $u_n > 0$ }

$$ma = m\frac{d^2u_n}{dt^2} = K(u_{n+1} - u_n) - K(u_n - u_{n-1})$$

$$d^2u_n/dt^2 = \frac{K}{m}(u_{n+1} - 2u_n + u_{n-1})$$

} Should look familiar!

Real chain of atoms ≡ Approximate wave equation

Sound Demo

Need:

Mac, running MacCRO (Cathode Ray Oscilloscope?)
Tuning Fork, medium
Organ Pipe, medium
Cardboard tube "Pipette"
Sonometer (string)

Input Settings: Set Gain 110

Voice:

Oscilloscope:
Time Scale 10 ms/div
Trace A 50 mV/div
Triggering
Deep voice: hum, show periodicity in wave form

Tuning Fork (medium):

Time Scale 0.1 ms/div
Trace A 10 mV/div
Show Sinusoidal wave form

Spectrum Analyzer:
Resolution 2 Hz
0-2000
0-5
Label Peaks
See single harmonic

Voice:

0-500
0-5
Freeze Display
See all the harmonics

Medium Pipe Organ (ask for volunteers):

0-2000
0-5
See all harmonics

Pipette:

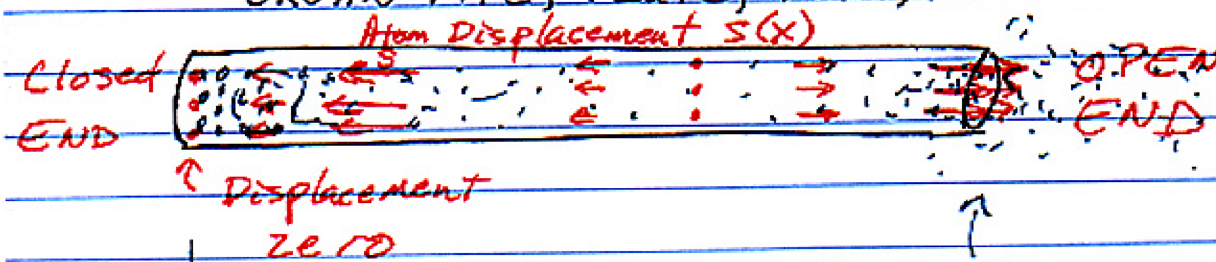
0-1000
0-5
See odd peaks high

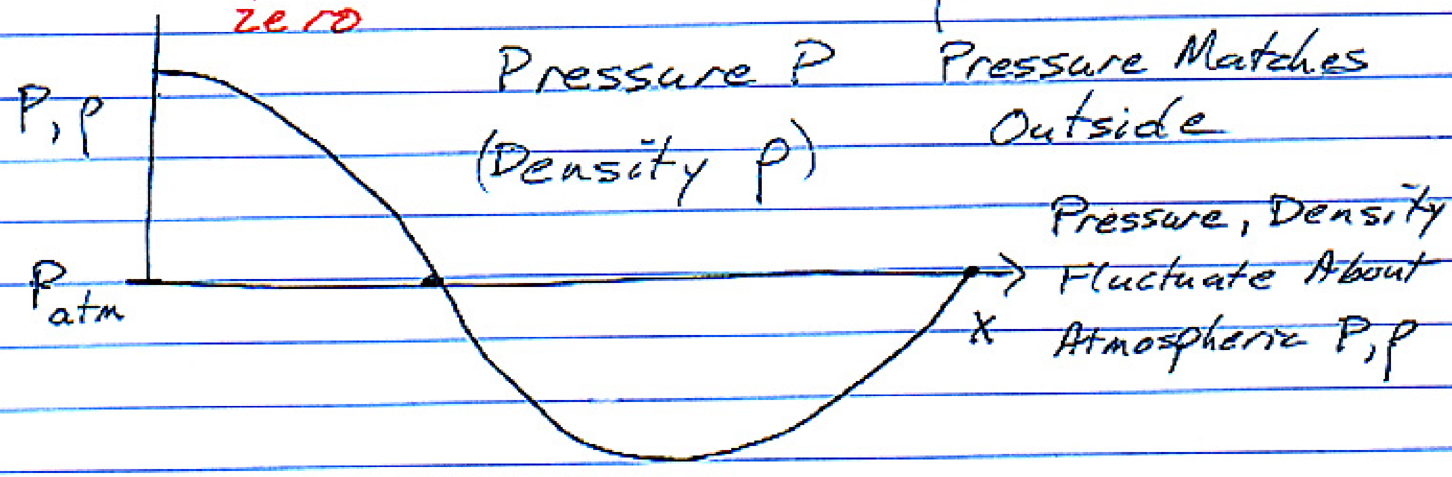
Sonometer:

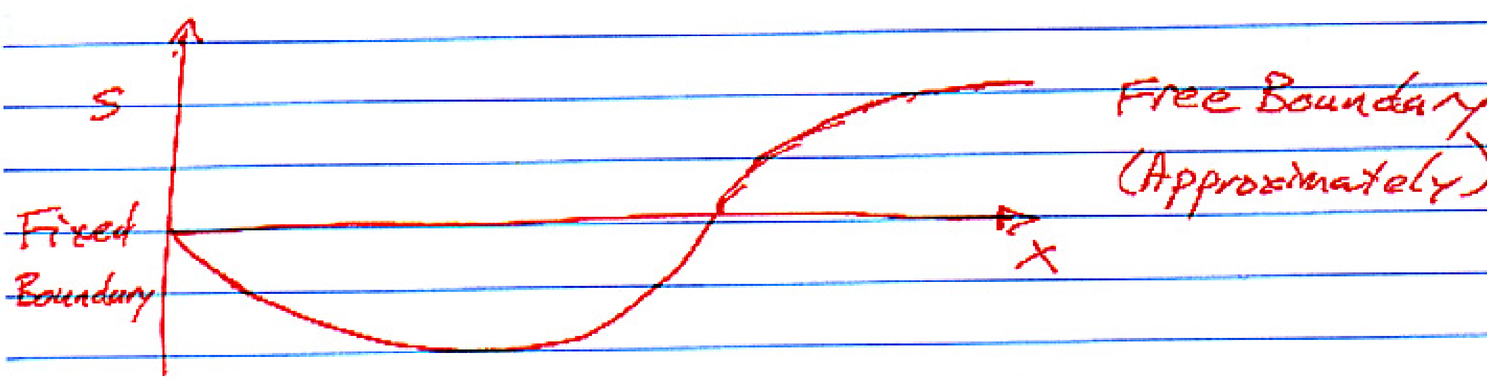
0-500
0-2
Pluck middle, odd harmonics (node at center for even harmonics)
Pluck near end, all harmonics

SOUND

DEMO: ORGAN PIPES
OSCILLOSCOPE & MIKE — Shape of wave
TUNING FORK — ⇕
FREQUENCY ANALYZER — Harmonics
ORGAN PIPE, FLUTE, TUBA, ...





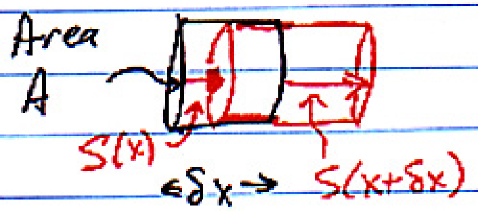


Wave Equation for longitudinal Sound in One Dimension

Air, water, solids: ($\lambda \gg a$, spacing between atoms) Pressure depends on Volume

$$P = P_0 - B\left(\frac{\Delta V}{V}\right)$$

B = Bulk Modulus
Good for small $\frac{\Delta V}{V}$



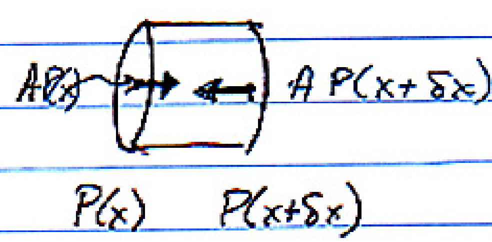
$$V = A\,\delta x$$

$$V + \Delta V = A\{\delta x + s(x+\delta x) - s(x)\}$$

$$\Delta V = A[s(x+\delta x) - s(x)]$$

$$P - P_0 = -B\frac{\Delta V}{V} = -B\frac{\cancel{A}[s(x+\delta x) - s(x)]}{\cancel{A}\,\delta x} = -B\frac{\partial s}{\partial x}$$

Pressure is Force per unit Area



$$\text{Force} = AP(x) - AP(x+\delta x)$$

$$= \underbrace{m}_{\rho A \delta x}\ \underbrace{a}_{\frac{\partial^2 s}{\partial t^2}}$$

$$\rho \cancel{A}\,\delta x \frac{\partial^2 s}{\partial t^2} = \cancel{A}(P(x) - P(x+\delta x))$$

$$\frac{\partial^2 s}{\partial t^2} = \frac{1}{\rho}\frac{P(x) - P(x+\delta x)}{\delta x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}$$

$$\frac{\partial P}{\partial x} = -B\frac{\partial^2 s}{\partial x^2}$$

Wave Equation for Sound

$$\boxed{\frac{\partial^2 s}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}}$$

Velocity of Sound $= \sqrt{B/\rho}$

in Air, 20°C

$= 343\ m/s \sim 1/5$ mile/~~hour~~ S

What's the Pressure for Traveling Sinusoidal Wave?

$$s(x,t) = S_{max}\cos\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$$P - P_0 = -B\frac{\partial s}{\partial x} = \underbrace{\frac{2\pi B}{\lambda} S_{max}}_{P_{max}} \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

What's the Kinetic Energy Density?

$$\frac{\text{Kinetic Energy}}{\text{Volume}} = \frac{\frac{1}{2}(M)\left(\frac{\partial s}{\partial t}\right)^2}{\underbrace{(V)}_{=\rho}} = \frac{1}{2}\rho\left(\frac{\partial s}{\partial t}\right)^2$$

Potential Energy = Kinetic Energy for Travelling Wave

$$\text{Total Energy Density} = \rho\left(\frac{\partial s}{\partial t}\right)^2$$

$$= \rho\, s_{max}^2 (2\pi f)^2 \sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

What's the <u>Intensity</u> of a traveling sound wave?

$$\text{Intensity} = \text{Power}/\text{Area}$$
$$= (\text{Energy Density}) \times \text{Velocity}$$
$$= \rho \left(\frac{\partial s}{\partial t}\right)^2 v = \rho\sqrt{B/\rho}\left(\frac{\partial s}{\partial t}\right)^2$$

$$I = \sqrt{B\rho}\left(\frac{\partial s}{\partial t}\right)^2 = \sqrt{B\rho}\,(2\pi f)^2 s_{max}^2 \sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

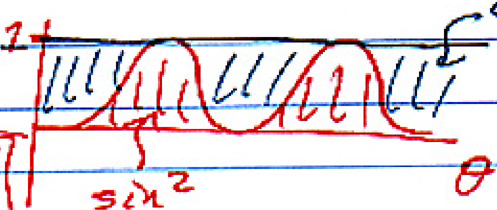
What's the <u>Average</u> Intensity?

Useful Trick: Average of $\sin^2$ is $\frac{1}{2}$

$\sin^2 + \cos^2 = 1$

average $\sin^2 = \frac{1}{2}$
average $\cos^2 = \frac{1}{2}$

$\boxed{\text{Integer \# of half-periods}}$



$$\text{Average Intensity} = \sqrt{B\rho}\,(2\pi f)^2 s_{max}^2 / 2$$
$\leftarrow$ $\sin^2$ average

[Express in terms of P_{max}^2]

$\boxed{\text{Units}}$ Intensity = Joules/sec per unit area
= Watts/m^2

At 1000 Hz, you can hear $I_0 = 10^{-12}\ W/m^2$ = 1 dB
corresponding to air displacing $s_{max} = 10^{-11}\ m \sim \frac{1}{30}$ atom$^{\circ}_{\circ}$
A power mower $I = 10^{-2}\ W/m^2$ = ten Giga (I_0)? Use Log Scale

Decibels $= \beta = 10 \log_{10}(I/I_0)$

Lawn Mower $= 10^{10} I_0 = 100$ dB