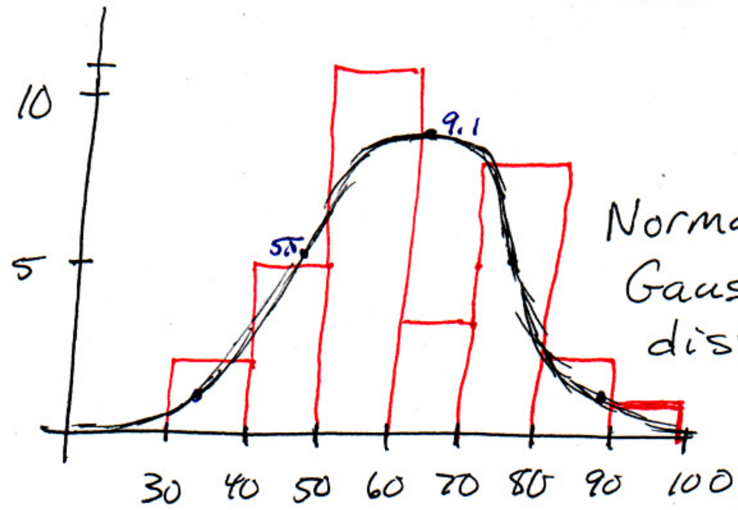
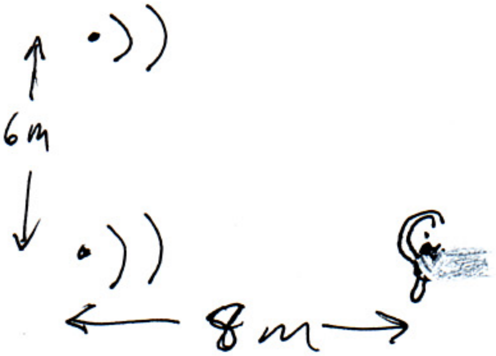
Mean: 61
σ : 14



Normal, or Gaussian distribution

Problem M1:



- HW 7, Sound Wave Interference

- $\Delta L = \sqrt{6^2+8^2} - 8 = 2\,m$

$= \lambda/2$ [Destructive]

$\omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{\pi v}{\Delta L} =$

$= \frac{340\pi}{2} = 170\pi$

- Amplitudes Add

$E_{tot} = \frac{E}{8} - \frac{E}{10}$; $I \sim E^2$

$I_{av}/I_8 = \left(\frac{E}{8} - \frac{E}{10}\right)^2 / (E/8)^2 \sim \left(\frac{1}{8} - \frac{1}{10}\right)^2 / \frac{1}{8^2} = \left(1 - \frac{8}{10}\right)^2$

$= \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

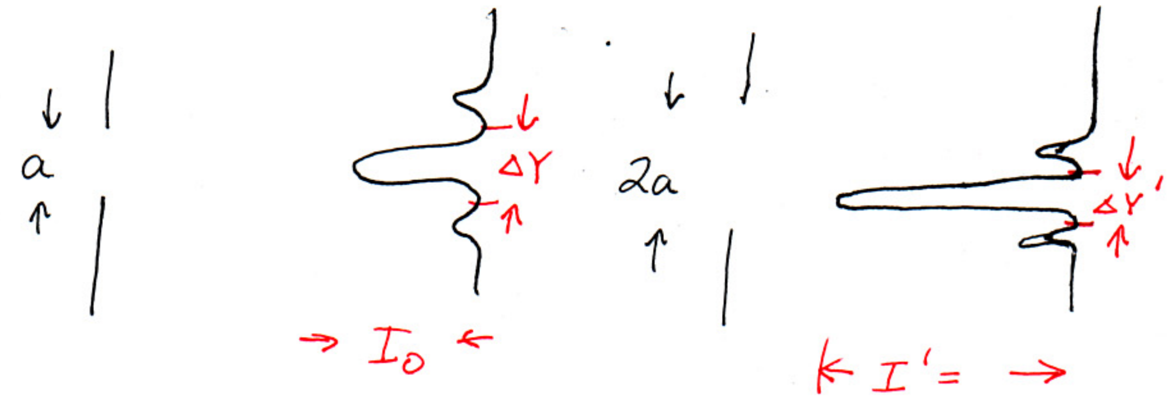
M2: $(\nabla \times A)^2 = \varepsilon_{ijk}\, \partial_j A_k\, \varepsilon_{ilm}\, \partial_l A_m$

$$= (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl})(\partial_j A_k)(\partial_l A_m)$$

$$= (\partial_j A_k)^2 - (\partial_j A_k\, \partial_k A_j)$$

Most people guessed a correct answer; many got the correct answer.

M3:



Maximum

- Amplitudes Add
- At $\theta = 0$, constructive interference

$$E' = 2E_0 \Rightarrow I' = 4I_0$$

Zeros (Related to uncertainty principle $\Delta x \Delta p \gtrsim \hbar$)

- $I = I_{max} \dfrac{\sin^2\alpha}{\alpha}$ zero at $\alpha = \pm\pi$

$\alpha = \dfrac{ak\sin\theta}{2} \sim \dfrac{ak\theta}{2}$ zero at $\theta = \pm\dfrac{2\pi}{ak}$

$a \sim \Delta x$ $\quad \Delta a \Delta\theta \sim \frac{1}{k}$
$\Delta\theta \sim \Delta p$

$\sim \dfrac{ak}{2}\dfrac{\Delta Y}{L}$ $\qquad \theta' = \pm\dfrac{2\pi}{(2a)k} = \theta/2 \Rightarrow \Delta Y' \stackrel{!}{=} \dfrac{\Delta Y}{2}$

- Rayleigh criterion, $\Delta\theta \sim \dfrac{\lambda}{a} = \dfrac{2\pi}{ak}$ for double slit
Resolving Power

M4: Elastic Wave

Transverse $c_T = \sqrt{\mu/\rho}$

$$\vec{u}(0,y,z,t) = (f(t), \overbrace{g(t), h(t)}^{\text{Transverse}})$$

Longitudinal $c_L = \sqrt{\frac{\lambda+2\mu}{L}}$

$$u_x(x,t) = f(\underbrace{t - x/c_L}_{(x-c_L t)(-\frac{1}{c_L})})$$

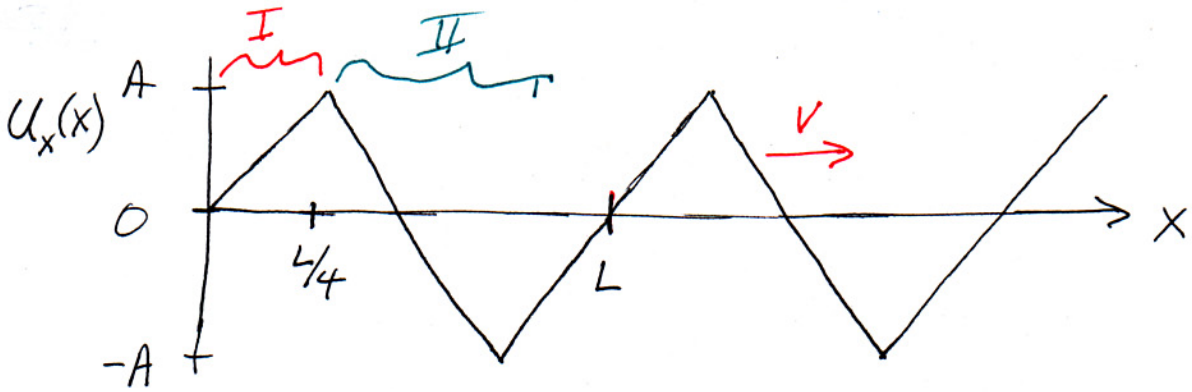
← agrees with $f(t)$ at $x=0$

traveling right

$$\vec{u} = \left(f(t-x/c_L),\ g(t-x/c_T),\ h(t-x/c_T)\right)$$

(Much like problem set 9, E&M travelling wave pulse).

S1: Sawtooth Wave



(S1.A) Intensity

$\left(\frac{\text{energy}}{\text{vol}}\right)\frac{\text{length}}{(\text{time})} = \frac{\text{energy}}{\text{area}\cdot\text{time}}$ $\quad \frac{\partial E}{\partial t} = \nabla\cdot I$ more-or-less

$$I = p\,\frac{\partial u}{\partial t} \quad \left[\text{or } I = EV, \text{ but not } I = \partial E/\partial t\,!\right]$$

$$p = -B\,\nabla\cdot u = -B\,\partial u/\partial x = -B\frac{A}{L/4} = -B\frac{4A}{L} \quad \text{Region I}$$

$$= +B\frac{A}{L/4} = B\frac{4A}{L} \quad \text{Region II}$$

$\frac{\partial u}{\partial t}$ Is Not V! (Chunk Velocity Is Not Sound Velocity!)

$$u(x,t) = u(x - vt) \qquad \frac{\partial u}{\partial t} = -v\,\partial u/\partial x$$

$$\partial u/\partial t = \begin{cases} -v\frac{4A}{L} & \text{Region I} \\ v\frac{4A}{L} & \text{Region II}\end{cases}$$

$$I = Bv\left(\frac{4A}{L}\right)^2 \quad \text{everywhere.}$$

(Review from Prelim I).

(S1.B) Sine wave intensity

$$u_n(x,t) = a_n \sin(k_n(x - vt))$$

$$I = p\,\partial u/\partial t \qquad \partial u/\partial t = -a_n k_n v \cos(k_n(x - vt))$$

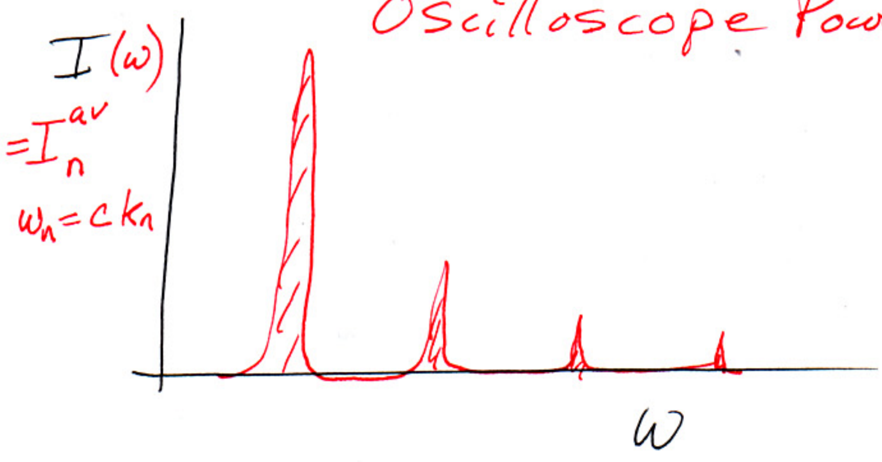
$$p = -B\,\nabla \cdot u = -B\,\partial u/\partial x = -B k_n \cos(k_n(x - vt))$$

$$I_n = B a_n^2 k_n^2 v \underbrace{\cos^2(k_n(x - vt))}_{\text{average} = 1/2}$$

$$\boxed{I_n^{av} = \frac{1}{2} B a_n^2 k_n^2 v}$$

I_n^{av} = "Intensity in channel #n"

Oscilloscope Power Spectrum



(51.C) $u = \sum a_n \sin(k_n(x - vt))$

$$a_n = (-1)^{(n-1)/2}\, 8A/\pi^2 n^2 \qquad n \text{ odd.}$$

Why should $I^{(av)} = \sum I_n^{(av)}$? {Crackling Paper Problem}

$$I = +P\,\partial u/\partial t$$

$$= \left(-B\,\partial u/\partial x\right)\left(-v\,\partial u/\partial x\right)$$

$$= Bv \sum_n a_n k_n \cos(k_n(x-vt)) \sum_m a_m k_m \cos(k_m(x-vt))$$

$$I_{av} = \sum_n \sum_m B v\, a_m a_n k_m k_n \underbrace{\frac{1}{T}\int_0^T \cos(k_n(x-vt)) \cos(k_m(x-vt))\,dt}_{\text{1/2 for } n=m,\ \text{zero if } n\neq m \text{ "Orthonormality" of cosines}}$$

$$= \sum_n \tfrac{1}{2} B v\, a_n^2 k_n^2$$

$$= \sum_n I_n^{av}$$

Does it work?

$$a_n^2 = \frac{64A^2}{\pi^4 n^4} \qquad k_n^2 = \frac{4\pi^2 n^2}{L^2} \qquad n \text{ odd}$$

$$\sum_{n\ \text{odd}} \frac{1}{2} BV \left(\frac{8A}{\pi^2 n^2}\right)^2 \left(\frac{2\pi n}{L}\right)^2 = \text{Constant}\left(\sum_{n\ \text{odd}} \frac{1}{n^2}\right)$$

$$= 128\, BV \left(\frac{A}{L}\right)^2 \frac{1}{\pi^2} \underbrace{\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots\right)}_{\pi^2/8}$$

$$= BV \left(\frac{4A}{L}\right)^2 \checkmark$$

S2. Waves on a Thin Wire

Most people got the strain field ε_{ij}.

(S2.A) $$\varepsilon_{ij} = \begin{pmatrix} Ak\cos kx & \frac{\sigma' A k^2 y \sin kx}{2} & 0 \\ \frac{\sigma' A k^2 y \sin kx}{2} & -\sigma' Ak\cos kx & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(S2.B) $$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\,\delta_{ij}$$

$$\varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = Ak\cos kx\,(1-\sigma')$$

$$\lambda\,\varepsilon_{kk}\,\delta_{ij} = \begin{pmatrix} \lambda(1-\sigma') & 0 & 0 \\ 0 & \lambda(1-\sigma') & 0 \\ 0 & 0 & \lambda(1-\sigma') \end{pmatrix} Ak\cos kx$$

Many people decided $\varepsilon_{kk}\delta_{ij} = \begin{Bmatrix} \varepsilon_{ij} & i=j \\ 0 & i\neq j \end{Bmatrix}$?!?

$$\sigma_{ij} = \begin{pmatrix} [2\mu+\lambda(1-\sigma')]\,Ak\cos kx & \mu\sigma' A k^2 y \sin kx & 0 \\ \mu\sigma' A k^2 y\sin kx & [-2\mu\sigma' + \lambda(1-\sigma')]\,Ak\cos kx & 0 \\ 0 & 0 & \lambda(1-\sigma')\,Ak\cos kx \end{pmatrix}$$

Amusing fact:

Thin wire <u>actually</u> deforms

$$u = A \begin{pmatrix} \left(1 - \sigma \frac{k^2 R^2}{2}\right) \sin(kx - \omega t) \\ -\sigma k y \cos(kx - \omega t) \\ -\sigma k z \cos(kx - \omega t) \end{pmatrix} + O(k^3)$$

Two afternoons work —
problem set next year?