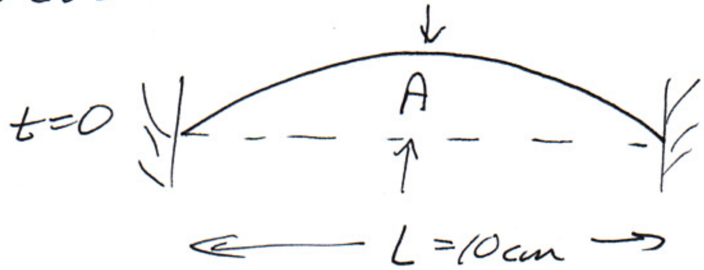
Exercise:



$\tau = 16$ dynes
$\lambda_0 = 1$ gm/cm

(a) What's ω? $\quad \lambda = 2L \quad k = \frac{2\pi}{\lambda} = \frac{\pi}{L} \quad \omega = ck = c\frac{\pi}{L}$

$c = \sqrt{\tau/\lambda_0} = 4$ cm/s $\quad \omega = c\frac{\pi}{L} = \frac{4\pi}{10} = 1.2567$ rad/s

(b) What's $y(x,t)$, in terms of k, ω, and A?

$y(x,t) = A \sin kx \cos \omega t$

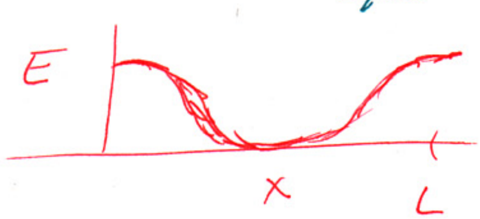
(c) What's $E(x,t)$, the total energy density? Plot $t=0$, $\omega t = \frac{\pi}{2}$. Is energy conserved?

$$E(x,t) = \frac{1}{2}\lambda_0\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}\tau\left(\frac{\partial y}{\partial x}\right)^2$$

$$= \frac{1}{2}\lambda_0\omega^2 A^2 \sin^2 kx \sin^2 \omega t + \frac{1}{2}\tau k^2 A^2 \cos^2 kx \cos^2 \omega t$$

Equal

$t=0$



$\omega t = \frac{\pi}{2}$

(d) What's $P(x,t)$, the power? Plot $t=0$, $\omega t = \frac{\pi}{2}$, $\omega t = \frac{\pi}{4}$, $\omega t = \frac{3\pi}{4}$

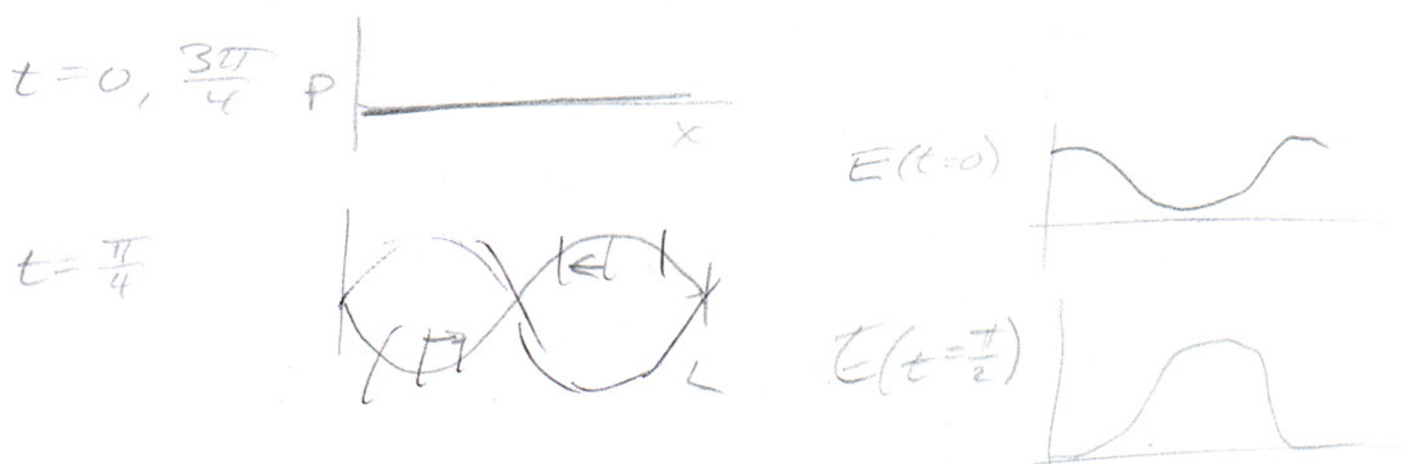
$$P(x,t) = -\tau \, \partial y / \partial x \; \partial y / \partial t$$

$$= -\tau (Ak) \cos kx \cos \omega t \, (-A\omega) \sin kx \sin \omega t$$

$$= A^2 \tau k \omega \cos kx \sin kx \cos \omega t \sin \omega t$$

$$\cos x \sin x = \tfrac{1}{2} \sin(2x) \qquad \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= A^2 \tau k \omega \sin(2kx) \sin(2\omega t)$$



(e) Show that energy is locally conserved.

$$\frac{\partial E}{\partial t} = -\frac{\partial P}{\partial x} \; ?$$

$$\tfrac{1}{2} \lambda_0 \omega^2 A^2 \sin^2 kx \, \underbrace{(2\omega \sin \omega t \cos \omega t)}_{\frac{1}{2}\sin 2\omega t} + \tfrac{1}{2} \tau k^2 A^2 \cos^2 kx \, \underbrace{(-2\omega \sin \omega t \cos \omega t)}_{\frac{1}{2}\sin 2\omega t}$$

$$\stackrel{?}{=} -2k A^2 \tau k \omega \underbrace{\cos(2kx)}_{\cos^2 kx - \sin^2 kx} \sin 2\omega t \qquad \lambda_0 \omega^3 = \tau k^2 \omega = \tau k^2 \omega$$

Exercise:

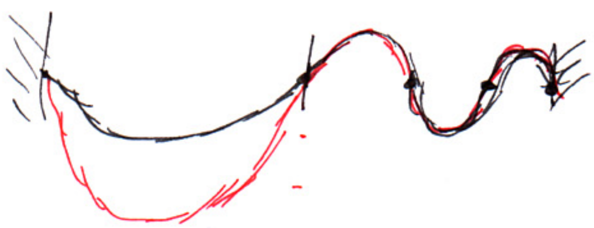
(a) Draw the lowest standing-wave eigenmode with a node at $x=0$

$$c_1 = \sqrt{T/\lambda_1} \quad c_2 = \sqrt{T/9\lambda_1}$$

$$c_1 = 3c_2$$

$$\omega_1 = \omega_2 \rightarrow k_1 = \frac{\omega_1}{c_1} = \frac{1}{3}\frac{\omega_2}{c_2} = \frac{1}{3}k_2$$

$\rightarrow$ (wavelength)$_1$ = 3 (wavelength)$_2$



(b) Write a formula for it.

$$x<0: \eta_1(x,t) = A_1 \sin(k_1 x) \qquad k_1 \frac{L}{2} = \pi \quad k_1 = \frac{2\pi}{L}$$

$$= A_1 \sin\left(\frac{2\pi}{L}x\right)$$

$$x>0: \eta_2(x,t) = A_2 \sin\left(\frac{6\pi}{L}x\right) \qquad k_2 = 3k_1$$

$$\frac{\partial \eta_2}{\partial x} = \frac{6\pi}{L}A_2 = \frac{\partial \eta_1}{\partial x} = \frac{2\pi}{L}A_1$$

$$3A_2 = A_1$$