Material for Week 3
Physics 4488/6562: Statistical Mechanics
https://sethna.lassp.cornell.edu/Teaching/562/
Exercises due Mon. Feb 10
Last correction at January 16, 2025, 6:35 pm

All exercises are from the second edition of the text: https://sethna.lassp.cornell.edu/StatMech/EntropyOrderParametersComplexity20.pdf

Monday
In-class question: 3.16 *Taste, smell, and μ*
Wednesday
Read: Chapter 4, Sec. 4.1 (Liouville's theorem), Sec. 4.2 (Ergodicity)
Pre-class question: 4.6 *Perverse initial conditions*
In-class question: 4.2 *Liouville vs. the damped pendulum*
In-class question: 3.11 *Maxwell relations*
Friday
Read: Chapter 5, Sec. 5.1 (Engines & Heat Death)
Pre-class question: 4.5 *No Hamiltonian attractors*
In-class question: 5.1 *Life and the heat death of the Universe*
Monday
Read: Chapter 5, Sec. 5.2.1 (Entropy of mixing)
Pre-class question: 3.18 *Ideal gas glass*

Assigned exercise for everyone

5.8 *The Arnol'd cat map.* (Mathematics, Dynamical systems) Cut-and-paste ergodicity. A chaotic Schrödinger's cat. Liouville's theorem for maps. Equilibration from chaos.

Special topic exercises (6562 do one; 4488 do 7 during 14 weeks)

3.14 *Pendulum energy shell.* The surface area of the energy surface is not equally occupied! You need the energy shell.

4.8 *Jarzynski.* Exact results in *non-equilibrium* statistical mechanics! Liouville's theorem applies also to time-dependent Hamiltonians. Jarzynski, and later Crooks, used this to calculate the exact entropy change for a non-equilibrium process. Here we use an ideal gas, compressed non-adiabatically, to illustrate how this exact result is used in practice.

4.9 *2D turbulence and Jupiter's great red spot.* (Astrophysics, Computation, Dynamical systems) Statistical mechanics with many conserved quantities (not just the energy). Two-dimensional turbulence explored in a vortex simulation

4.3 *Invariant measures.* (Mathematics, Complexity, Computation, Dynamical systems) Liouville-like analysis of dissipative chaos. Dissipative dynamical systems have an 'invariant measure' that generalizes the phase-space averages justified by Liouville's theorem. Here we apply this to a chaotic, one-dimensional map exhibiting the period-doubling route to chaos.
Hints at https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html

4.7 *Crooks.* Proving exact results for non-equilibrium systems. Here we derive the nonequilibrium Crooks relation using Liouville's theorem.