

Material for Week 9

Physics 4488/6562: Statistical Mechanics

<https://sethna.lassp.cornell.edu/Teaching/562/>

Exercises due Mon. Mar 24

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The exercises with numbers N1.xxx are to be found in <https://sethna.lassp.cornell.edu/StatMech/SethnaExercises.pdf>

For next Monday's pre-class question 9.2, do parts (a) and (b) only.

Monday

In-class question: [8.19](#) *2D Ising cluster expansions*

Wednesday

Read: Chapter 9, Sec. 9.1 (Broken symmetry) and 9.2 (Order parameter)

Pre-class question: [9.9](#) *Ising order parameter*

In-class question: [9.10](#) *Nematic order parameter*

Friday

Read: Chapter 9, Sec. 9.3 (Examine the elementary excitations)

Pre-class question: [9.15](#) *Superfluid second sound*

In-class question: [9.6](#) *Symmetries and wave equations*

Monday

Read: Chapter 9, Sec. 9.4 (Classify the topological defects)

Pre-class question: [9.2](#) *XY defects*

Assigned exercise for everyone

- 8.6 *Metropolis*. (Mathematics, Computation) The second-simplest, and most common Monte-Carlo method
- 8.8 *Wolff*. (Mathematics, Computation) The most exotic and efficient Monte-Carlo algorithm. Cluster flips satisfying detailed balance!

Special topic exercises (6562 do one; 4488 do 7 during 14 weeks)

- 8.21 *Fruit flies and Markov*. (Biology) Fly behavior and Markov chains (inspired by Gordon Berman's experiments, using an older machine learning technique).
- 9.5 *Landau theory for the Ising model*. (Condensed matter) Another kind of order parameter, again using symmetries to derive the emergent laws (here the free energy). Commonly used to study phase diagrams, defects, and boundary conditions. An uncontrolled approximation unless fluctuations are added. Gives correct behavior near continuous phase transitions only for systems with long-range interactions or in high spatial dimensions.
- N4.50 *Localization*. (Quantum, Condensed matter) Doped insulators and the metal-insulator transition.
- 9.12 *Rigidity of crystals*. (Order parameters) Crystals flow under stress too. How are they different from liquids? The first parts are fundamental; the later sections estimate how unimportant the flow is using diffusion.
- 9.14 *Sound and Goldstone's theorem*. (Condensed matter) Why long-wavelength fluctuations have low frequencies
- 9.20 *Number and phase in superfluids*. (Quantum) An example of a powerful method for deriving equations of motion from commutation relations and Poisson brackets (i.e., beyond just using symmetry)
- 8.23 *Kinetic proofreading in cells*. (Biology) How cells violate detailed balance to replicate DNA without errors. This is a classic result in biology, perhaps with deep links to topological insulators.
- 8.22 *Metastability and Markov*. Arrhenius barrier crossing as a Markov process. Prelude to Exercise 12.22. Hints at <https://sethna.lasp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>
- N1.29 *Rubber band dynamics I: Random walk*. Rubber band as a random walk. Links to the infinite-range Ising model. Hints at <https://sethna.lasp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>
- N1.30 *Rubber band dynamics II: Diffusion*. Rubber band as a diffusion equation. Links to the infinite-range Ising model. Hints at <https://sethna.lasp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>