Material for Week 11

Physics 4488/6562: Statistical Mechanics

https://sethna.lassp.cornell.edu/Teaching/562/

Exercises due Mon. Apr 14

Last correction at January 16, 2025, 6:36 pm

Calculus of variations is needed for Wednesday's pre-class question, analogous to the derivation of the Euler-Lagrange equations in mechanics The variational derivative is the first term in a 'function-space' Taylor series of $\mathcal{F}[\rho+\delta]-\mathcal{F}[\rho]$ with respect to $\delta(x)$.

Monday

In-class question: 10.13 *Onsager regression hypothesis*

Wednesday

Read: Chapter 10, Sec. 10.5 (Susceptibility and linear response) and 10.6 (Dissipation and the imaginary part)

Pre-class question: 10.14 *Liquid dynamics*

In-class question: 10.15 *Harmonic susceptibility, dissipation*

Friday

Read: Chapter 10, Secs. 10.7 (Static susceptibility), 10.8 (The fluctuation-dissipation theorem), and 10.9 (Causality and Kramers-Krönig)

Pre-class question: 10.16 *Harmonic fluctuation-dissipation*

Pre-class question: 10.18 *Harmonic Kramers–Kronig*

In-class question: 10.17 *Susceptibilities and correlations*

Monday

Read: Chapter 11, Sec. 11.1 (Stable and metastable phases) and 11.2 (Maxwell construction)

Pre-class question: 11.11 *Unstable to what?*

Assigned exercise for everyone

10.9 *Quasiparticle poles and Goldstone's theorem.* (Condensed matter) Poles in the susceptibility describe quasiparticles and their lifetimes.

Special topic exercises (6562 do one; 4488 do 7 during 14 weeks)

10.1 *Cosmic microwave background radiation.* (Astrophysics) The cosmic microwave background radiation is the echo of the Big Bang. It has amazing information about the evolution of the Universe and the laws and parameters that govern it (like the dark matter and dark energy content). This information is gleaned from the angular temperature-temperature correlation function. The most striking feature is a striking oscillation, due to 'acoustic' waves in the early universe. We analyze a wave equation describing these oscillations, and calculate the predicted oscillations.

10.4 *Spin.* (Condensed matter) Dynamics of a gyroscope! Linear response for a spin in a thermal environment

10.5 *Telegraph noise in nanojunctions.* (Condensed matter) We examine the time-time current correlations through nanojunctions, which arise from the hopping of individual atoms. If you found the derivation in section 10.4 (Onsager regression hypothesis) daunting, this exercise is for you.

10.8 *Magnet dynamics.* (Condensed matter) The next step after fluids: $M(x,t)$.

10.19 *Critical point response.* Linear response near a critical point (see Exercise 12.27). A preview of the renormalization-group scaling we study in Chapter 12.

N1.15 *Cell signaling and mutual information.* (Biology, Statistics) Mutual information is somewhat analogous to entropy, and is a widely useful measure. We use it to describe hungry E.-coli bacteria. In our model, cells maximize the mutual information by evolving signal pathways from membrane receptors sensing food to flagella moving them forward.