Material for Week 13
Physics 4488/6562: Statistical Mechanics
https://sethna.lassp.cornell.edu/Teaching/562/
Exercises due Mon. Apr 28
Last correction at January 16, 2025, 6:37 pm

For Wednesday's pre-class question, do part (a) only. We'll do the other parts in class.

The exercises with numbers N1.xxx are to be found in https://sethna.lassp.cornell.edu/StatMech/SethnaExercises.pdf

Monday
In-class question: 12.1 *Ising self-similarity*
In-class question: 12.15 *Hearing chaos*
In-class question: 12.14 *Crackling noises*
Wednesday
Read: Chapter 12, Sec. 12.1 (Universality)
Pre-class question: 12.7 *Renormalization-group trajectories*
In-class question: 12.7 *Renormalization-group trajectories*
Friday
Read: Chapter 12, Sec. 12.2 (Scale Invariance)
Pre-class question: 12.3 *Scaling and coarsening*
In-class question: 12.8 *Superconductivity and the renormalization group*
Monday
Read: Chapter 12, Sec. 12.3 (Examples of critical points)
Pre-class question: 12.16 *Period doubling and the onset of chaos*

Assigned exercise for everyone

12.11 *RG and the central limit theorem: long.* (Mathematics) Remember random walks produce Gaussians? Here's an RG derivation. Adding random numbers as a renormalization group.

Special topic exercises (6562 do one; 4488 do 7 during 14 weeks)

12.9 *Period doubling and the RG.* . (Mathematics, Complexity, Computation, Dynamical systems) The onset of chaos and the RG. You'll reproduce Feigenbaum's original analysis of the period doubling onset of chaos. Hints at https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html

N1.33 *RG flows from non-perturbative coarse graining.* A real-world 3D Ising renormalization-group calculation. RG flows, critical exponents, exponent relations, ... Hints at https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html

N1.28 *Ising critical correlations.* Ising model correlation functions near the critical point.

N1.27 *Coarsening correlations.* Ising model correlation functions during coarsening.

12.4 *Bifurcation theory.* (Dynamical systems) Universality classes for differential equations near qualitative changes in behavior ('phase transitions').

12.5 *Mean-field theory.* (Condensed-matter) Magnets in dimensions bigger than four are simpler. Assuming a spin feels the average magnetization of its neighbors gives the correct exponents above four dimensions. See also 12.26 and 12.27.

12.26 *Ising mean field derivation.* Derive a mean-field theory for the Ising model in a field, using the rigorous mean-field bound of Exercise 12.27.

12.27 *Mean-field bound for free energy.* Proof, due to Gibbs, Bogoliubov, and Feynman, that the mean-field calculation in Exercise 12.26 is a rigorous upper bound for the free energy.

12.28 *Avalanche size distribution.* Deriving the universal scaling function for the avalanche size distribution in a mean-field theory (probably valid for dimensions larger than six).