

## Material for Week 14

Physics 4488/6562: Statistical Mechanics

<https://sethna.lassp.cornell.edu/Teaching/562/>

Exercises due Mon. May 05

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This homework is due next Monday, the last day of class.

The final exam will be distributed on Monday May 6, just after the the last day of class, and is due at 10am on Monday May 13, one week later.

The exam this year will have two exercises focused on the renormalization group; Exercise N2.1, now assigned as the common exercise for the last day of class, is quite relevant for both exercises. Two of the exercises have a numerical component. Please be sure beforehand to figure out how to generate PDFs from notebooks, either for individual figures (to incorporate into your answers) or for the notebook as a whole. Hints will be given in Mathematica and Python, so expertise in numerically solving ODEs and finding eigenvalues should not be crucial.

### Monday

In-class question: [12.29](#) *The onset of chaos: lowest order RG*

### Wednesday

Read: ‘Crackling crossover’, James P. Sethna, *Nature Physics*, **3**, 518-9 (2007)

Pre-class question: [12.2](#) *Scaling and corrections to scaling*

In-class question: [12.31](#) *Singular corrections to scaling*

### Friday

Read: ‘Crackling wires’, James P. Sethna, *Science*, **318**, 207-8 (2007)

Pre-class question: [12.19](#) *Diffusion equation and universal scaling functions*

In-class question: [12.21](#) *Earthquakes and wires*

### Monday

Read: No reading for today

Pre-class question: [12.18](#) *Random walks and universal exponents*

In-class question: [N1.19](#) *Beer*

## Assigned exercise for everyone

12.33 *Pandemic.* (Epidemiology, Computation) RG for disease outbreaks. Hints available.

## Special topic exercises (6562 do one; 4488 do 7 during 14 weeks)

12.32 *Conformal invariance.* (Mathematics, Biology, Computation) Critical points have amazing symmetries. The Ising model in two dimensions at  $T_c$  can be warped by any complex analytic function and still looks like an Ising critical point. Hints at <https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>

N1.7 *Beer and rigidity: Jamming.* Bear foam is a solid! Can we use RG to predict its properties?

12.30 *The onset of chaos: full RG.* (Computation, Dynamical systems) Here we implement Feigenbaum's numerical calculation of the universal scaling function for period doubling, and his amazingly high-precision extraction of the 'critical exponents'  $\alpha$  and  $\delta$ . Hints at <https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>

12.25 *Critical correlations.* Qualitative comparison of an Ising simulation at its critical point with a space-time correlation function introduced in Exercise 10.19

12.22 *Activated rates and the saddle-node transition.* (Computation) Chemical reaction rates form a universal scaling function for barrier crossing! RG treatment of Arrhenius barrier crossing rates for small barriers (see Exercise 8.22). Hints available.

12.6 *The onset of lasing.* (Quantum, Optics, Mathematics) Universal fluctuations in a non-equilibrium transition

12.23 *Biggest of bunch: Gumbel.* (Mathematics, Statistics, Computation, Engineering) What is the probability distribution for the largest flood of the century (ignoring global warming)? Hints at <https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>

12.24 *Extreme values: Gumbel, Weibull, and Fréchet.* (Mathematics, Statistics, Engineering) The RG derivation for the biggest or smallest of  $N$  random numbers. Rare events in floods, insurance, mechanical failure. . .