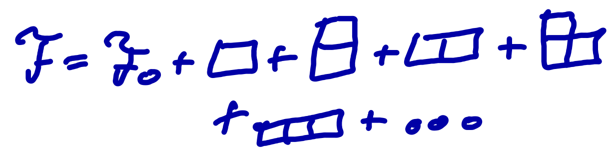
Linked cluster expansion

How connected clusters exponentiate to fill space.

- One spin flip at origin, $Z = Z_0(1 + \mathrm{e}^{-\delta/k_BT})$.
- m dilute spins scattered at random, no collisions

$$
\begin{aligned}
Z &= Z_0(1 + \cdots + N(N-1)\ldots(N-m)\mathrm{e}^{-m\delta/k_BT}) + \ldots \\
&\approx Z_0(1 + \cdots + (N^m/m!)\mathrm{e}^{-m\delta/k_BT}) + \ldots \\
&= Z_0 \sum (N\mathrm{e}^{-\delta/k_BT})^m/m! \\
&= Z_0 \exp(N\mathrm{e}^{-\delta/k_BT}).
\end{aligned}
$$



  exponentiates to fill space.

- Free energy $F = -k_BT \log Z = F_0 - Nk_BT \exp(-\delta/k_BT)$
- Correction per spin given by one spin flip at origin
- Next correction given by cluster of two touching spins at origin
- Linked cluster theorem. Feynman diagrams. Localization…