Eigenvectors, left and right

Non-symmetric and non-Hermitian matrices are *weird*.

- Left eigenvectors and right eigenvectors
- Right eigenvectors not orthonormal: coefs from left
- Many purposes better served by *Singular values* σ_n
  - Given by eigenvalues of $A^T A$: $\sigma = \sqrt{\lambda_{A^T A}}$
  - Generalizes eigs: for symmetric A, $\sigma_n = |\lambda_n|$
  - Also works for non-square matrices.
  - PCA, SVD numerical more stable, ...
- Markov equilibration rate $\leq$ rate of slowest eig
  - $\rho_n - \rho^* = \sum_{\alpha=1} \lambda_\alpha^n a_\alpha \rho_\alpha \gtrsim \lambda_1^n$ at long times
  - Can be longer. Markov chain $x \to x+1$ length N

$$\begin{pmatrix} 0 & 0 & 0 & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & & & & \dots \\ 0 & 1 & 0 & & & & \dots \\ \dots & 0 & 1 & 0 & & & \dots \\ & \dots & & & & & \\ & & \dots & 0 & 1 & 0 & 0 \\ & & & \dots & 0 & 1 & 1 \end{pmatrix}$$

    takes N steps to fall off cliff, but all eigenvalues are zero except bottom of cliff

- Left & right eigenvectors also important in renormalization group (Chapter 12).