

Material for Week 3

Physics 4488/6562: Statistical Mechanics

<https://sethna.lassp.cornell.edu/Teaching/562/>

Exercises due Mon. Feb 13

Last correction at December 22, 2022, 2:08 pm

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All exercises are from the second edition of the text: <https://sethna.lassp.cornell.edu/StatMech/EntropyOrderParametersComplexity20.pdf>

Monday

In-class question: 3.16 *Taste, smell, and μ*

Wednesday

Read: Chapter 4, Sec. 4.1 (Liouville's theorem), Sec. 4.2 (Ergodicity)

Pre-class question: 4.6 *Perverse initial conditions*

In-class question: 4.2 *Liouville vs. the damped pendulum*

In-class question: 3.11 *Maxwell relations*

Friday

Read: Chapter 5, Sec. 5.1 (Engines & Heat Death)

Pre-class question: 4.5 *No Hamiltonian attractors*

In-class question: 5.1 *Life and the heat death of the Universe*

Monday

Read: Chapter 5, Sec. 5.2.1 (Entropy of mixing)

Pre-class question: 3.18 *Ideal gas glass*

Exercises for everyone

5.8 *The Arnol'd cat map.* (Mathematics, Dynamical systems) Cut-and-paste ergodicity

Select one (4488) or two (6562)

3.14 *Pendulum energy shell.* Surface area is not microcanonical

5.22 *The Dyson sphere.* (Astrophysics) Entropy, energy, and advanced civilizations.

4.8 *Jarzynski.* Liouville's theorem applies also to time-dependent Hamiltonians. Jarzynski, and later Crooks, used this to calculate the exact entropy change for a non-equilibrium process. Here we use an ideal gas, compressed non-adiabatically, to illustrate how this exact result is used in practice.

4.9 *2D turbulence and Jupiter's great red spot.* (Astrophysics, Computation, Dynamical systems) Two-dimensional turbulence explored in a vortex simulation

4.3 *Invariant measures.* (Mathematics, Complexity, Computation, Dynamical systems) Dissipative dynamical systems have an 'invariant measure' that generalizes the phase-space averages justified by Liouville's theorem. Here we apply this to a chaotic, one-dimensional map exhibiting the period-doubling route to chaos. Hints at <https://sethna.lassp.cornell.edu/StatMech/EOPCHintsAndMaterials.html>

4.7 *Crooks.* Here we derive the nonequilibrium Crooks relation using Liouville's theorem.