STATISTICAL PHYSICS
Physics 7653, Fall 2018

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We will focus on *renormalization-group* methods for explaining *emergent scale invariance*.

What is scale invariance?

System looks 'the same' on different scales (up to overall magnitudes)

Continuum limit:

- Liquids, gases (Navier Stokes) — Boring on
- Solids (Elasticity theory) — all long scales

Quantum electrodynamics — Fluctuations on all scales

Continuous Phase Transitions

- Ising model at T_c — Fluctuations
- 2D Melting — on all scales
- XY, Heisenberg, Potts, … — at transition
- Spin glass, Metal-insulator, …

Onset of Chaos — Events on all time scales

Avalanches and Crackling Noise — Events of all sizes

What does *emergent* mean?

Striking regularity happens (emerges) for non-obvious reasons from complex microscopic behavior

(elastic constants, viscosities / power laws, scaling)

What is the renormalization group?

- Invented at Cornell
  - Ken Wilson, Michael Fisher, Leo Kadanoff ~1970
  - Wilson Nobel prize
- Amazing use of abstraction: coarse graining in "system space"
- Most qualitative results accessible without messy calculations
- Calculating exponents demands amazingly weird

Amazing amount of physics best described by RG

Prerequisites:

Statistical mechanics (one semester graduate course: partition functions, free energies, correlation functions)

Desire to understand deep underpinnings of subject

Texts:

John Cardy, *Scaling and Renormalization in Statistical Physics* (first half of course)

JPS, *Statistical Mechanics: Entropy, Order Parameters and Complexity* (Edition 2: v2 on line, esp. chapter 12)

First half: *flipped format.*

- Reading before each class
- Pre-class question each Thursday (due 8:30am)
- Homework on Tuesday
- Class activities: work alone / collaborate / consolidation
- Research shows this cycle optimizes learning. Cornell physics intro classes have converted to this format:
- 1σ gain in learning.
- As future faculty, your opportunity to see the method as participants
- Common knowledge: most learning done in HW; HW in groups better. Why not HW in groups guided by prof?
- Professionals work in groups, read papers, ask questions
- Still exploring best practices. Feedback welcome. (Morning pre-class submission time; consolidation)

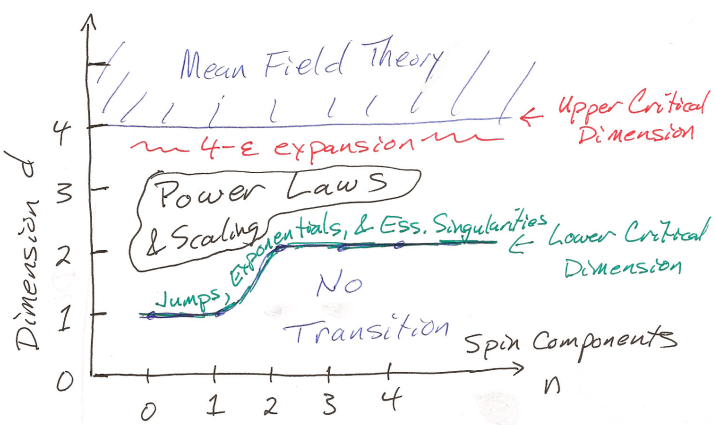
Second half: projects (see list of references)

- Conformal field theory visualization (2008, Machta), EOPC exercise 12.32
- Mosh pits. 2010, Silverberg & Bierbaum, 0.7M views)
  http://prl.aps.org/abstract/PRL/v110/i22/e228701
  http://cohengroup.ccmr.cornell.edu/projects/collective-motion-mosh-pits
  http://mattbierbaum.github.io/moshpits.js
- Zombies (2010, Alemi & Bierbaum, 1.2M views)
  http://journals.aps.org/pre/abstract/10.1103/PhysRevE.92.052801
  http://mattbierbaum.github.io/zombies-usa/
- Chaos and RG (2017, class and Ragu)
  Reexamining the renormalization group: Period doubling onset of chaos
  https://arxiv.org/abs/1807.09517
- Chemical barrier crossing as RG scaling function (2017, class and Hathcock)

Standard RG Theory:
Continuous Phase Transitions

$$H = -\Sigma J \boldsymbol{S}_i \cdot \boldsymbol{S}_j - \Sigma \boldsymbol{H} \cdot \boldsymbol{S}_i$$

n-component spins $\boldsymbol{S}_i$; $\boldsymbol{S}_i^2 = 1$
Universality class from n and d
  n=1 Ising
  n=2 XY model
  n=3 Heisenberg
  n=0 Self-avoiding random walks!
dimension d
  d=1 linear chain
  d=2 surface, sheet
  d=3 bulk
High temperatures: Disordered
Low temperatures: Ordered
Transition at T_c , $\boldsymbol{H}$=0
(Also tricritical Ising, other universality classes)



Lower critical dimension d_l: Entropy wins over energy

- Excitations which destroy order, stiffness have more entropy than energy as size L diverges at all temperatures
- (For example, finite energy excitation domain wall in d=1 Ising model)
- Anomalous scaling behavior: logs, exponentially diverging correlation lengths, jumps in stiffness, essential singularities in specific heat

Upper critical dimension d_u: Fluctuations become unimportant

- High dimensions, many neighbor spins
- Each spin feels "average neighborhood" enough to not change critical exponents, qualitative behavior near T_c
- Logs at upper critical dimension

Calculational methods – exciting new developments

ε-expansion near upper critical dimension

- Usually uses Feynman diagrams analogous to those in quantum field theory
- Cardy uses Operator Product Expansion, analogous to tools used in string theory
- Logs at upper critical dimension

Conformal field theory in 2D

- Exact critical exponents; explains why 2D is solvable
- Scale invariance + rotations & translations = Conformal group = all analytic functions

Conformal bootstrap (Tom Hartman)

- Works in all dimensions (including fractional!)
- Gives critical exponents to high precision

Nonperturbative functional ('Exact') RG

- Good answers to traditional problems
- Only systematic approach without known point to perturb from
- Remarkable results in disordered systems, turbulence, maybe quantum gravity

Normal form theory (Our group; Raju, Hayden, Kent-Dobias, Liarte)

- Systematizes logs, exponentials, jumps, essential singularities into universality *families*
- Predicts variables for universal scaling functions
- Predicts corrections to scaling (extending the reach)