Mean Field Theory

Jim Sethna, Physics 7653, Fall 2018

Ignores fluctuations (or assumes they are short-range)

- Fluctuations on all scales near (many) critical points

Captures behavior in

- High spatial dimension > d_{UCD}
  - Tricritical Ising $d_{UCD} = 3$
  - Jamming $d_{UCD} = 2$
  - Percolation, Random-field Ising model $d_{UCD}=6$
- Infinite-range systems
- Long-range interactions
- Rough models for inhomogeneous systems, defects

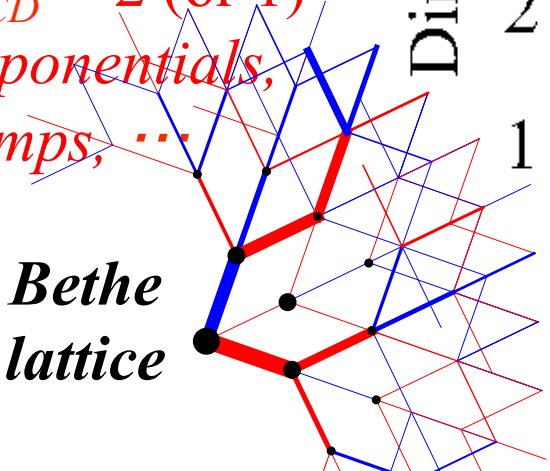
High dimensions

Mean-field theory above the *Upper Critical Dimension*

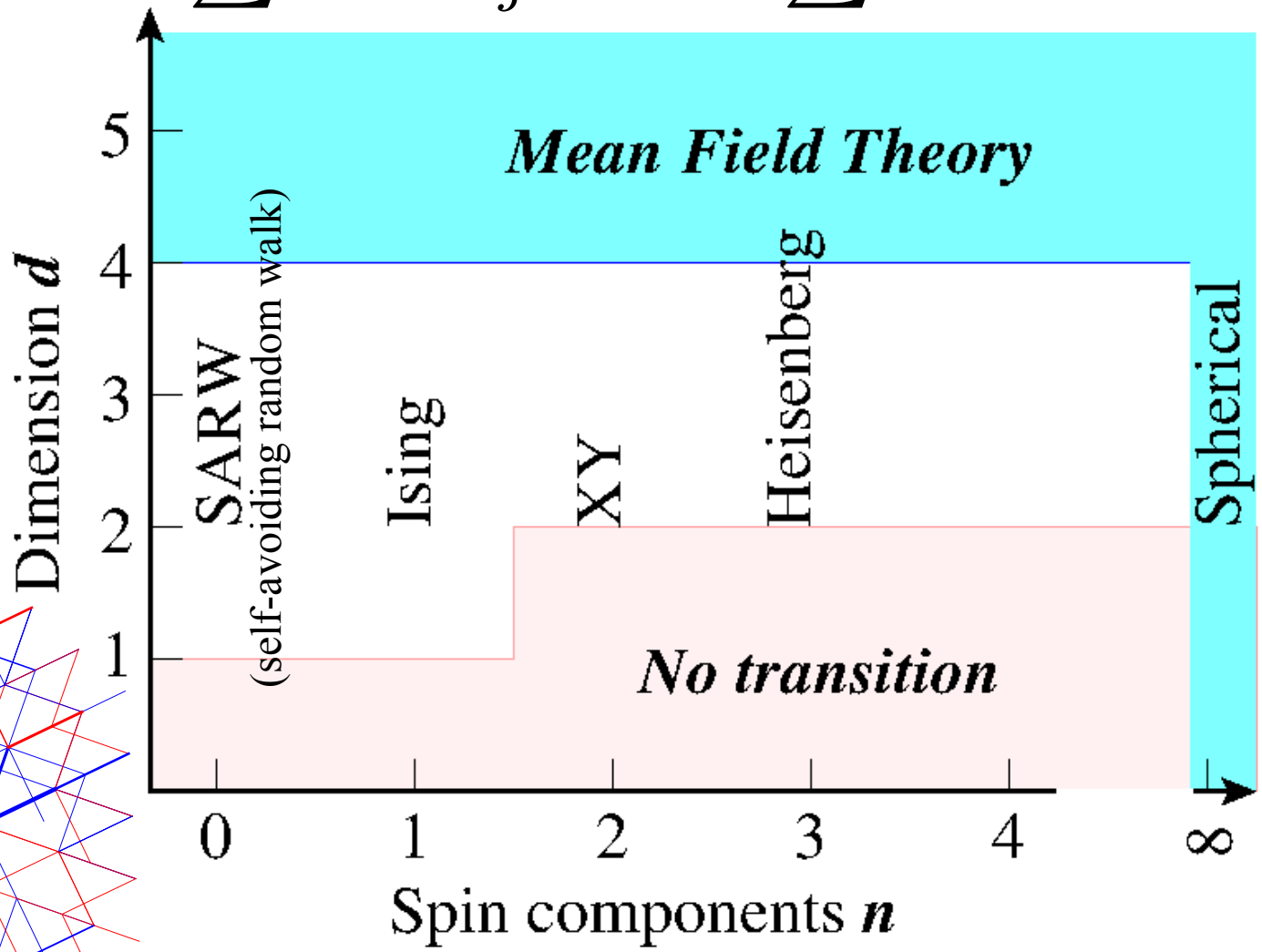
$$\mathcal{H} = -J \sum \mathbf{S}_i \cdot \mathbf{S}_j - \mathbf{H} \cdot \sum \mathbf{S}_i$$

High T: melted
Low T: ordered
Critical: T_c, $\boldsymbol{H=0}$

$d_{UCD} = 4$
4-ε expansion
logs, PowerLogs
$d_{LCD} = 2$ (or 1)
exponentials, jumps, …



Bethe lattice



Infinite-range interactions

Every site interacts with all/many others

Superconductors:

Cooper pairs overlap thousands of neighbors

Sync (Strogatz):

Every firefly sees all others

Laser onset criticality (Exercise 12.6):

Photons in nonlocal cavity modes

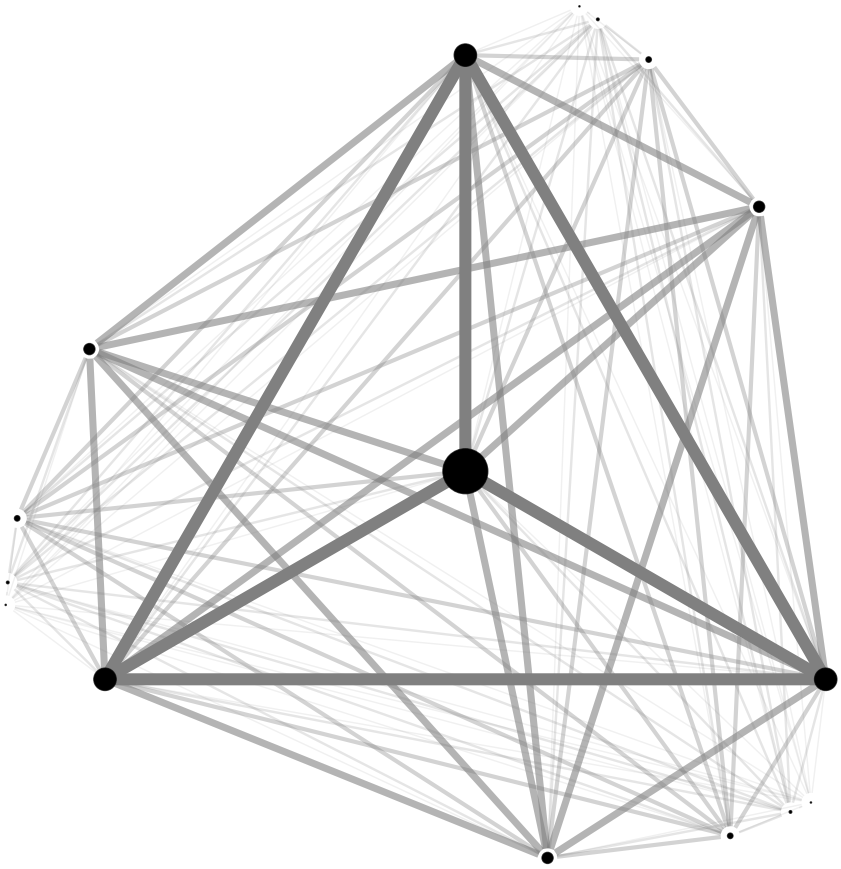
Neurons in brain

Axons long, 1000's of connections

Computer science (Selman and Gomes)

3SAT (Exercise 1.8), NP complete

Physics 'cavity' methods winners



Long-range interactions

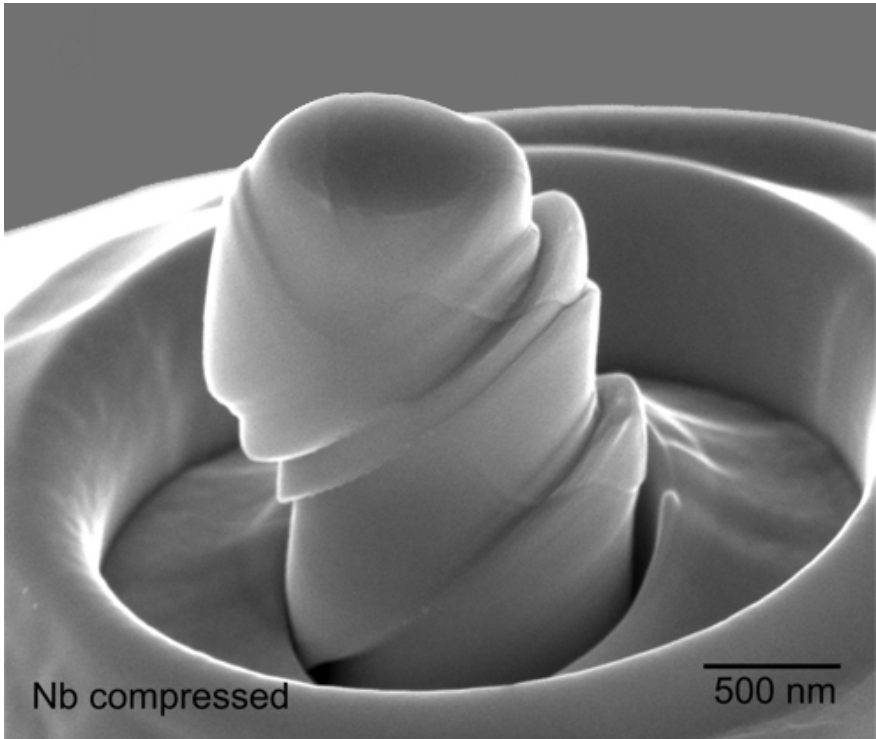
Power-law interactions between sites make d_{UCD}=3

Dipolar interactions between magnetic spins

- Not important near T_c for thermal transitions (M vanishes, screening)
- Important for noise in hysteresis (Barkhausen noise, avalanches)

Elastic interactions between slips

- Earthquake faults
- Dislocations in crystals (crackling noise)
- Shear transformation zones in amorphous metals



Rough guide to behavior

Mean-field theory as a practical, but uncontrolled, approximation

Curie-Weiss for ferromagnets (Cardy)
Van der Waals for liquids and gases
Sherrington-Kirkpatrick for spin glasses

- Ultrametricity
- Replica symmetry breaking
- Cavity method

Avalanche size distribution (Exercise 12.25)
Ginzburg-Landau theories (Exercise 9.5)

- Starting point for ε-expansions