Advanced topics on sloppy models for group projects
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Below find a few open research topics on sloppy models that could be used as ambitious class projects. Most of them have been looked at by others, and we would have to ask their permission before plunging in. On the other hand, they could act as useful sources and collaborators in the endeavor.

In rough order of effort and risk (*e.g.*, more people needed in the group), the projects are

1. **Visualization of Gaussian fits.** (Check with Katherine Quinn and Qingyang Xu.) A Gaussian or normal distribution fits the mean and standard deviation $\theta = (\mu, \sigma)$ to a sample – say heights from a sample of women, see Exercises 1.13 and 4.7. *(a) One can compute the Fisher Information Metric $g_{\mu\nu}$ for this distribution (Exercise 1.15), and use standard differential geometry to calculate the curvature. The manifold has constant negative curvature, and covers what is called the* Poincaré half plane. *(b) One can then use Quinn's intensive embedding method [2] to visualize this manifold in a Minkowski space, and contrast your results with that of the Hellinger sphere embedding (Exercise 1.16). (c) We have two open questions for this cool example. First, as the mean varies from $-\infty \to \infty$, the intensive embedding becomes very long in one direction. How does translational symmetry exhibit itself in the embedding? Second, there are hints on the Web that there is a known embedding of the Poincaré half-plane in a finite-dimensional Minkowski space (some kind of parabola), but we have not tracked it down. Finding the scalar curvature for conics in Minkowski space would be a natural first step.*

2. **Fisher information, curvature, and the renormalization group.** (Check with Katherine Quinn and Ruoshui Wang.) Quinn's isometric embedding of the Ising model [2] currently uses the entire probability distribution for spin configurations, and is thus confined to small systems (like 4×4). The model manifold is known to have a curvature singularity at the critical point. *(a) Use the fact that the Ising model is an exponential family in the variables $\theta = (\beta, \beta H)$, to write the Fisher Information Metric $g_{\mu\nu}$ in terms of derivatives of the free*

energy per spin (Exercise 6.23. (b) Use the universal scaling form for the free energy to calculate $g_{\mu\nu}$ and the scalar curvature R near the critical point. What universal power law describes the divergence of the curvature near T_c? We have explored various approximate free energies to examine this in terms of the universal scaling function. *(c) Use the Ising model mean-field theory (Exercise 12.5) to give an approximate form for $g_{\mu\nu}$ and the curvature in high dimensions. (Why is it approximate away from the critical point?) (d) Use Quinn's isometric embedding to visualize a neighborhood of the critical point.* So far, we find that the cusp at the critical point is not visible in the largest three principle components; it is buried in the 'thinner' directions of the model manifold. *(e) Explore rotations of the principle components (or, better, Lorentz transformations) to pull out the important direction that illuminates the cusp most naturally.*

3. **Geodesics and entropy growth.** (Check with Ben Machta, Katherine Quinn, and Colin Clement.) Machta [1] (Exercise 6.23) argues that there is an entropy cost needed to control the parameters in a thermodynamic system. For example, in the Carnot cycle of a piston there is a minimum cost for the outside world to drive a piston through the cycle of isothermal and adiabatic compression and expansion. Fortunately for refrigerators and heat engines, this extra cost is sub-extensive, growing as the square root of the number of gas atoms in the piston. However, this would seem to mess up Szilard's argument that information entropy and thermodynamic entropy can be exchanged – that one can burn information (Exercise 5.2). *(a) Machta's argument focuses on a grand canonical ensemble, controlling the chemical potential by exchanging particles with a series of baths. Flesh out the analogous argument needed to control the pressure (e.g., using variable transmissions). (b) When compressing the gas, the number of particles is held constant, with no additional cost. When adiabatically compressing the gas, the temperature and pressure both must change, so one would seem to need the geodesic distance for both (as assumed in Exercise 6.23). But holding the energy constant (as one does in adiabatic compression) would seem not to need a control cost. What is the minimum cost for control for the adiabatic compression?* It would seem important to check that the same cost for control happens in other machines designed to extract work out of information. *(c) The Jarzynski engine (video at* **`https:`**

//www.youtube.com/watch?v=0OTyIShzR6o, *paper at Physics Today* **67**, *8, 60 (2014); https://doi.org/10.1063/PT.3.2490) is a good starting point. Will stochastic height fluctuations allow the mass to fall? Does it fall faster than the information entropy pushes it to rise?*

References

[1] Machta, B. B. (2015). Dissipation bound for thermodynamic control. *Phys. Rev. Lett.*, **115**, 260603.

[2] Quinn, K. N., Bernardis, F. De, Niemack, M. D., and Sethna, J. P. (2017). Visualizing theory space: Isometric embedding of probabilistic predictions, from the Ising model to the cosmic microwave background. *http://arxiv.org/abs/1709.02000*.