

Ising Universality

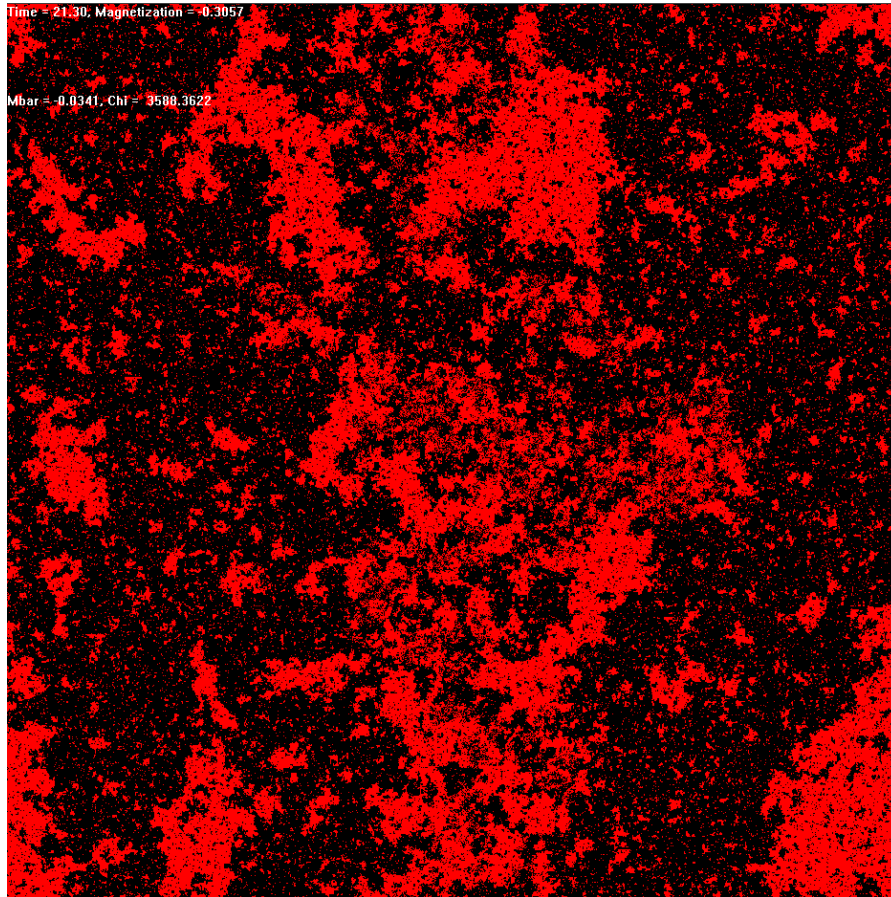
Jim Sethna, Physics 7653, Fall 2018

Ising universality class describes

- Magnets with uniaxial (up-down) asymmetry
- Disordered binary alloys
- Liquid-gas critical point
- Phase separation near end of coexistence line:
living cell membranes
- Quantum magnets (in one lower dimension)
- Nonequilibrium systems with two competing states
pistachio groves

The Ising Model at T_c

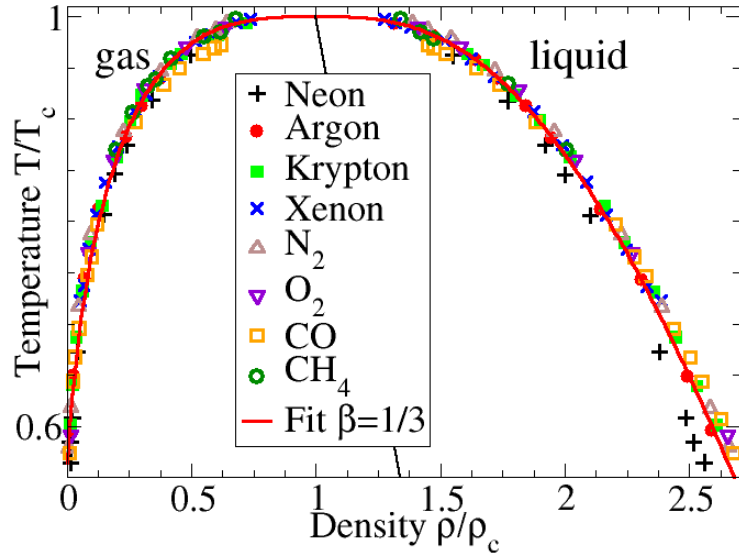
Structure on All Scales



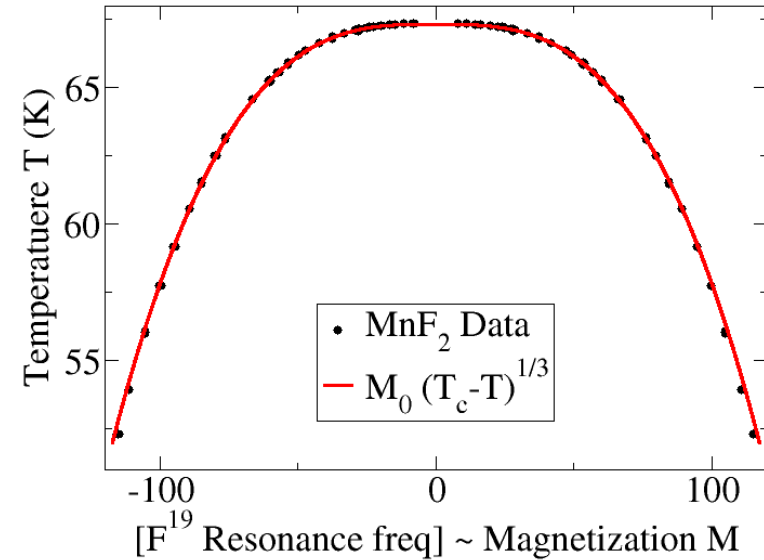
- Continuous Phase Transition
- Competition Entropy vs. Energy
- Thermal Disorder
- High Temperature:
Random
- Low Temperature
Long-Range Order
- Critical Point
 - $T_c = 2/\log(1+\sqrt{2}) \sim 2.27$
 - Fluctuations on All Scales

Universality: Shared Critical Behavior

Ising Model and Liquid-Gas Critical Point



Same critical
exponent
 $\beta=0.326!$



Liquid-Gas Critical Point

$$\rho - \rho_c \sim (T_c - T)^\beta$$

$$\rho^{Ar}(T) = \text{XXX} \rho^{CO}(\text{XXX})$$

Ising Critical Point

$$M(T) \sim (T_c - T)^\beta$$

$$\rho^{Ar}(T) = A(M(\text{XXX} T), T)$$

Universality: Same Behavior up to Change in Coordinates

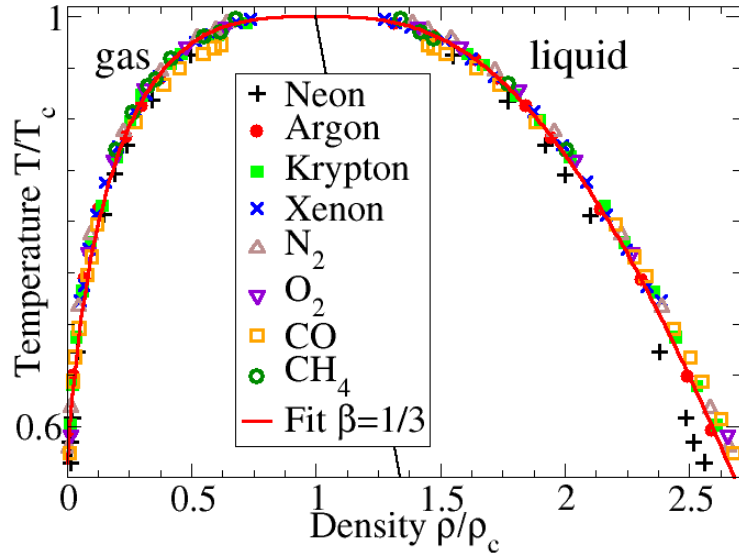
$$A(M, T) = \text{XXX}$$

Nonanalytic behavior at critical point (not parabolic top)

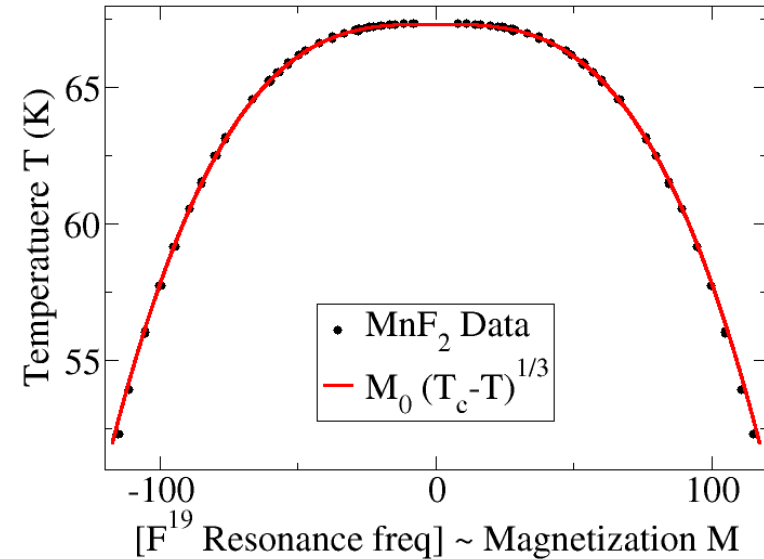
All power-law singularities (χ, c_v, ξ) are shared by magnets, liquid/gas

Universality: Shared Critical Behavior

Ising Model and Liquid-Gas Critical Point



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Liquid-Gas Critical Point

$$\rho - \rho_c \sim (T_c - T)^\beta$$

$$\rho^{Ar}(T) = A \rho^{CO}(BT)$$

Ising Critical Point

$$M(T) \sim (T_c - T)^\beta$$

$$\rho^{Ar}(T) = A(M(BT), T)$$

Universality: Same Behavior up to Change in Coordinates

$$A(M, T) = a_0 + a_1 (M - M_c) + a_2 (T - T_c) + \textit{(other singular terms)}$$

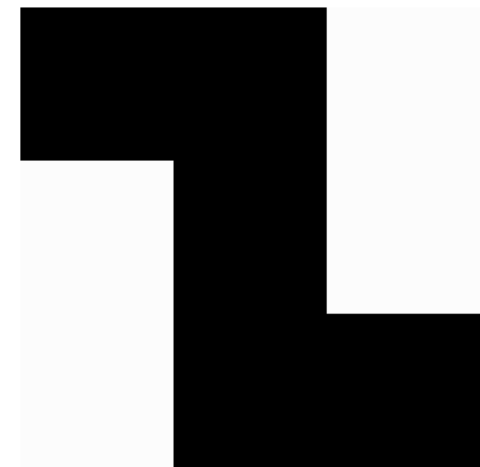
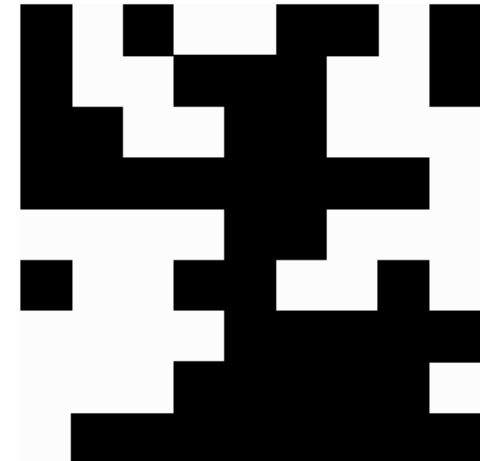
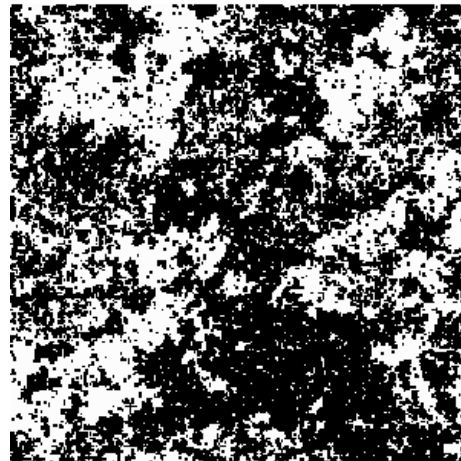
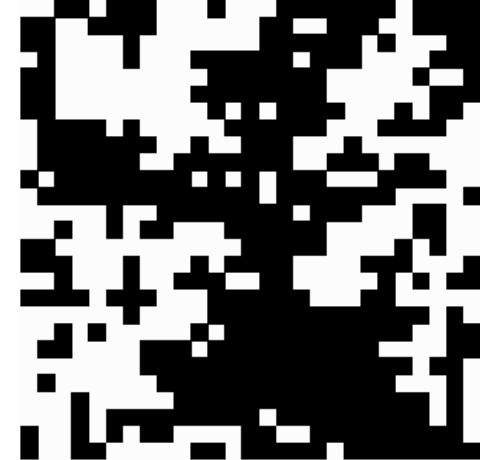
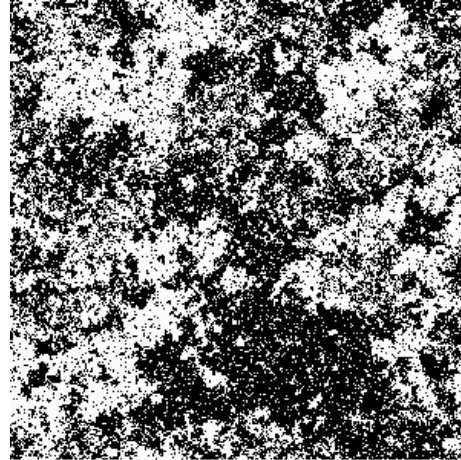
Nonanalytic behavior at critical point (not parabolic top)

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Coarse Graining

Remove microscopic details

- ‘Continuum limit’ – average over details in small regions, get effective laws for coarser system
- Example: majority-rule block-spin transformation (3x3 blocks)
- Renormalization group: find effective block-spin free energy: new interactions from old by tracing over microscopic variables
- Coarse-grain 1-D Ising?



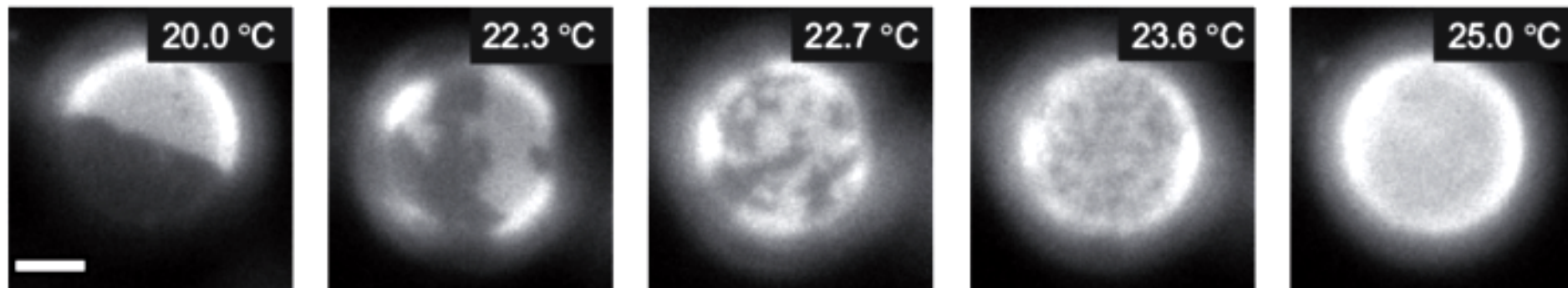
2D Ising Criticality in Cell Membranes

Sarah Veatch, Baird group (Chemistry)

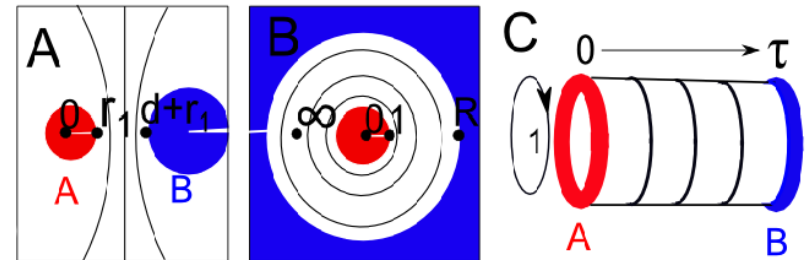
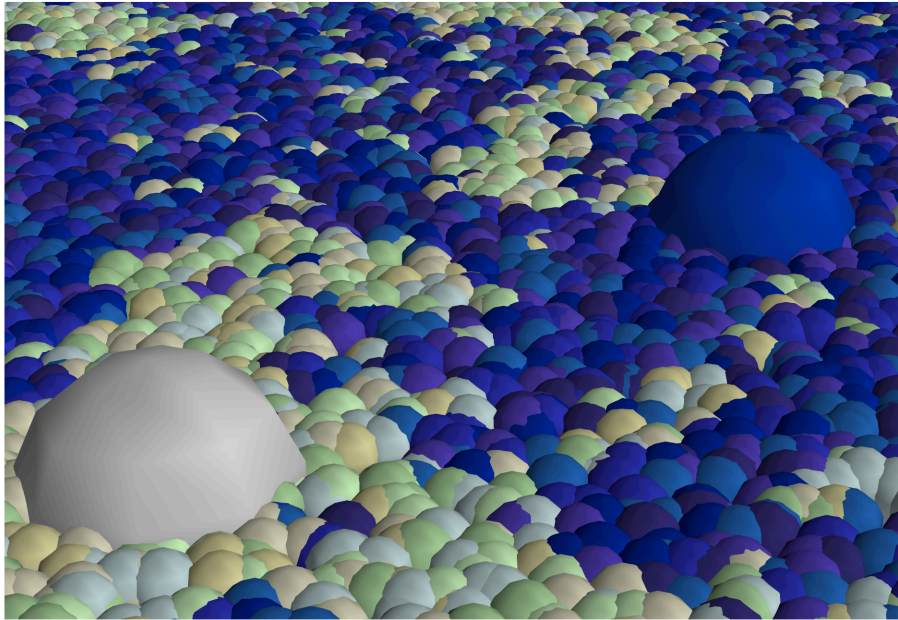
- Baird: Living cell membranes, when detached from the cell, undergo a phase separation near body temperature
- Sarah Veatch: This is a continuous transition in the Ising universality class.
- Connections with “lipid rafts”?

Hundreds of lipids;
thousands of membrane
proteins.

Separation into ‘floppy
tails’ and ‘stiff tails with
cholesterol’ = spin up &
spin down.



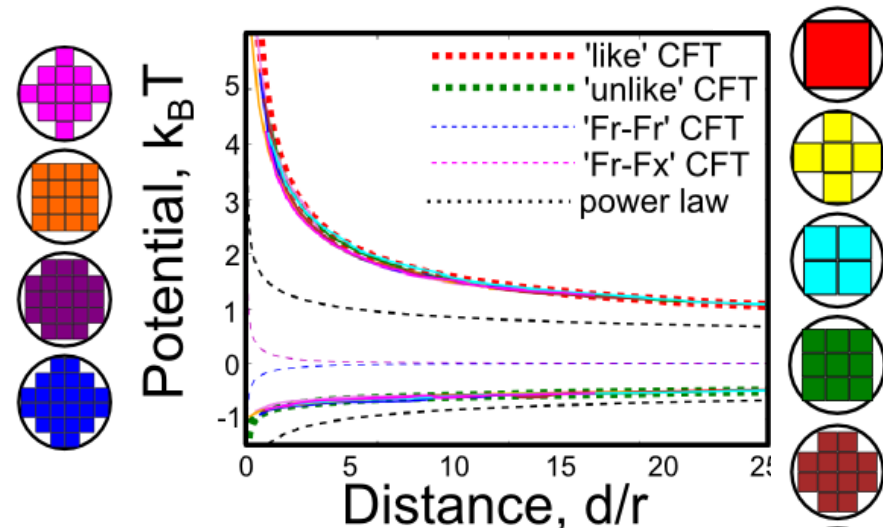
Using Ising fluctuations to communicate Long-range Casimir forces between proteins



Conformal field theory
(Dedekind and Jacobi elliptic
fcts) agrees with Monte Carlo

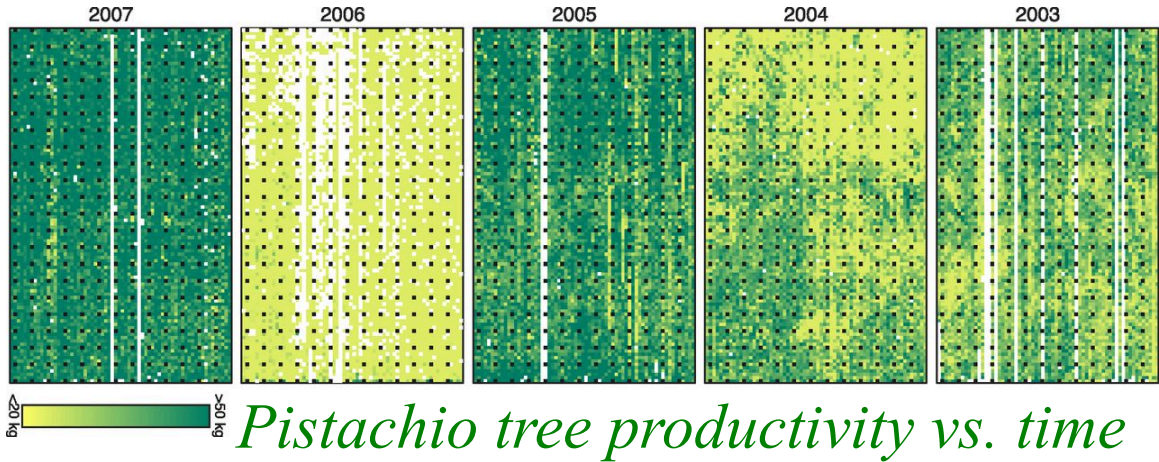
Proteins prefer 'up' or 'down'
Power-law lipid puddles
Puddle overlap gives attraction/
repulsion
Long range $r^{-1/4}$

Too weak to cluster; may partition



Ising Criticality in Fireflies, Pistachio Groves

Andrew Noble (grad Cornell, w/Strogatz fireflies)



No need for
microscopic systems,
equilibrium behavior!

Pistachio trees yield more in alternate years. They get into synchrony with their neighbors, showing Ising critical behavior.

