Physics 7653: Statistical Physics
http://www.physics.cornell.edu/sethna/teaching/653/
Material for Week 2
Exercises due Tuesday Sep 4
Last correction at August 25, 2018, 4:21 pm

Pre-class Preparation

Thursday

Read: Cardy, Chapter 2 (Mean Field Theory), Sections 2.1–2.2

Do: Review Exercise 9.5 (Landau free energy for the Ising model). Discuss the terms in eqn 9.19. At a non-zero magnetic field H, what term or terms would you add? Submit electronically by 8:30 Thursday morning.

Tuesday

Read: Cardy, Sections 2.3–2.4. Pay particular attention to the Ginzburg criterion – our excuse for using mean-field theory in high dimensions.

Mean Field: Introduction

Mean field theory can be derived and motivated in several ways.

1. The interaction (field) from the neighbors can be approximated by the average (mean) field of all sites in the system, effectively as described in Cardy section 2.1. This formulation makes it easy to understand why mean-field theory becomes more accurate in high dimensions and for long-range interactions, as a spin interacts with more and more neighbors.

2. The free energy can be bounded above by the free energy of a noninteracting mean-field model. This is based on a variational principle I first learned from Feynman (Exercise 12.24, based on Cardy's exercise 2.1).

3. The free energy can be approximated by the contribution of one order parameter configuration, ignoring fluctuations. (From a path-integral point of view, this is a 'zero loop' approximation.) For the Ising model, one needs first to change away from the Ising spin variables to some continuous order parameter, either by coarse-graining (as in Ginzburg-Landau theory, Sethna 9.5 below) or by introducing Lagrange multipliers (the Hubbard-Stratonovich transformation, not discussed here).

4. The Hamiltonian can be approximated by an infinite-range model, where each spin does interact with the average of all other spins. Instead of an approximate calculation

for the exact Hamiltonian, this is an exact calculation for an approximate Hamiltonian – and hence is guaranteed to be at least a sensible physical model. See Exercise 12.25 for an application to avalanche statistics.

5. The lattice of sites in the Hamiltonian can be approximated as a branching tree (removing the loops) called the *Bethe lattice* (not described here). This yields a different, solvable, mean-field theory, which ordinarily has the same mean-field critical exponents but different non-universal features.

Exercises

12.5: Mean-field theory.

12.23: Ising mean-field derivation.

12.24: Mean-field bound for free energy.