Physics 7653: Statistical Physics
http://www.physics.cornell.edu/sethna/teaching/653/
Material for Week 9
Exercises due Tuesday Oct. 31
Last correction at October 29, 2017, 2:01 pm

Pre-class Preparation

Thursday

Cardy, sections 4.2, 4.3, and 4.5.

High T_c, imaginary time, and the LCD. Many of the high-T_c superconductors are highly anisotropic, and are often modeled as two-dimensional layered quantum systems. Presume a theory with a quantum critical point. Note that, unlike Cardy's Fig. 4.5, there is no phase transition seen above the superconducting dome; it is more reminiscent of the one-dimensional transverse Ising chain, which has a quantum critical point at $T = 0$ with a crossover region for $T > 0$.
What might we presume about the lower critical dimension for the classical phase transition, to give a quantum critical point but no classical transition?
(Submit electronically by 9:30 Wednesday evening.)

Tuesday

Read: Power Spectra of Strange Attractors, B. A. Huberman and Albert B. Zisook, *Phys. Rev. Lett* **46**, 626 (1981), and Spectral Broadening of Period-Doubling Bifurcation Sequences, J. Doyne Farmer *Phys. Rev. Lett.* **47**, 179 (1981).

Noise and period doubling (Artem Bolshakov). Beyond the period-doubling critical point, broad-spectrum noise doesn't immediately drown out all of the periodic behavior. Instead, we see a multitude of chaotic "bands", which merge together into one. If we were to decrease μ while approaching the critical point, these bands would split up in much the same way that the single fixed point split into orbits of length 2^n, as you can see below. Within these bands, the logistic map is "semiperiodic": given x_0, we don't know the exact value of x_k, but all the values x_0, x_1, $x_2 \ldots x_{2^n}$ fall into a predetermined relative order. Thus, we can think of the action of f as periodic with a stochastic noise term. It turns out, the strength of this noise grows by a specific value beta between each bifurcation. The paper above discusses the growth of this noise with μ.
Why do you think that the band merging of the inverse cascades has the same δ as the "forward" bifurcation we have already studied? In the forward cascade, the stable fixed point of f^{2^n} always split into an orbit of two because a specific condition was met – the derivative at that point surpassed 1 in absolute value.
Why do you think that f^{2^n} stops mapping a specific band of values into itself past a certain specific value of μ? Hint: what happens to the entire logistic map past $\mu = 1$?
(Submit electronically to Artem Bolshakov (a.t.bolsh@gmail.com) by 9:30 Monday evening; cc Sethna.)

Exercises

No exercises this week. Focus on preparing for presentations...

References

[1] Feigenbaum, Mitchell (1979, 12). The universal metric properties of nonlinear transformations. *Journal of Statistical Physics*, **21**, 669–706.

[2] Fisher, Michael E. and Randeria, Mohit (1986, May). Location of renormalization-group fixed points. *Phys. Rev. Lett.*, **56**, 2332–2332.

[3] Mao, Jian-min and Hu, Bambi (1987). Corrections to scaling for period doubling. *Journal of Statistical Physics*, **46**, 111.

[4] Ostlund, S., Rand, D., Sethna, J. P., and Siggia, E. D. (1983). Universal properties of the transition from quasi-periodicity to chaos in dissipative systems. *Physica D*, **8**, 303–342.

[5] Rand, David, Ostlund, Stellan, Sethna, James, and Siggia, Eric D. (1982, Jul). Universal transition from quasiperiodicity to chaos in dissipative systems. *Phys. Rev. Lett.*, **49**, 132–135.

[6] Sethna, J. P. and Myers, C. R. (2004). *Entropy, Order Parameters, and Complexity* computer exercises: Hints and software. http://www.physics.cornell.edu/sethna/StatMech/ComputerExercises.html.