Physics 7653: Statistical Physics
http://www.physics.cornell.edu/sethna/teaching/653/
Material for Week 10
Exercises due Tuesday Nov. 7
Last correction at October 30, 2017, 9:57 pm

Pre-class Preparation

Thursday
Read:
EOPC Section 11.4.1 (Coarsening)
Coarsening dynamics of an isotropic ferromagnetic superfluid, Lewis A. Williamson, P. B. Blakie, https://arxiv.org/abs/1703.09360

1. **Coarsening in isolated quantum systems..**[1] ⓟ

   In EOPC, we saw that the evolution following a quench from disordered to ordered phase can be described by self similar behavior. The autocorrelation function $C(\vec{r}, t) = \langle \phi(\vec{r}, t)\phi(0, t)\rangle$ has universal form $C(\vec{r}, t) = f(|\vec{r}|/L(t))$, where t is the time after the quench. Here $L(t)$ is the typical length scale of the system and it grows as a power law in time (up to log correction) $L(t) \propto t^{1/z}$. Without conservation, length scale grows as $L(t) \propto t^{1/2}$. For scalar(Ising) model with conservation, length scale grows as $L(t) \propto t^{1/3}$. For vector mode ($O(N)$ with $N \geq 2$) with conservation, length scale grows as $L(t) \propto t^{1/4}$. However, the dynamics of the system can be affected by other slow modes [1].

   The system(spin 1 condensate in 2D) in [3] has a ferromagnetic phase and a polar phase as a function of control parameter q. The ferromagnetic phase has 3 types, easy-plane(XY), isotropic(Heisenberg), and easy-axis(Ising) as a function of q. The zero temperature quench from the polar to ferromagnetic phase turns out the have scaling exponents consistent with classical dynamical universality class. Note energy and magnetization along z are conserved.

   Ignoring logarithmic correction, sketch the dynamic exponent z (where $L(t) \sim t^{1/z}$) as a function of q. Here the magnetization along z is conserved, while that in x, y plane is not conserved, i.e., *the Ising 'easy axis' order evolves with conserved dynamics, while the 'easy plane' XY phase evolves with no conserved order parameter. If the order parameter conservation law and the dimension of order parameter are the only relevant parameters, what is the expected z in the Ising and XY case?* This should hint at the importance of other modes and conserved quantities in the system.

[1] Problem developed by Hil Fung Harry Cheung, 2017.

(Submit electronically to Harry Cheung (hc663@cornell.edu) by 9:30 Wednesday evening; cc Sethna.)

Tuesday

Read: Section 2.2 (pages 5 - 11) of Holger Gies, "Introduction to the functional RG and applications to gauge theories" (2006) https://arxiv.org/pdf/hep-ph/0611146.pdf. Section 2.1 might be helpful to get familiar with the used notation. While reading those sections, you don't have to focus too hard on the technical details. I will go over most of them in my presentation. More important, try to understand the idea of the flow equation for the effective action and how it is related to the calculations we have done in class. The role of the introduced regulator function is crucial.

If you are interested, the original paper, proposing the flow equation is: Christof Wetterich. Exact evolution equation for the effective potential. Phys. Lett., B301:90-94, 1993 http://www.sciencedirect.com/science/article/pii/037026939390726X?via%3Dihub

2. **FunctionalRG.**[2] ⓟ

   Consider eqn (28) in the paper. It describes the flow of the effective action, as more and more momentum shells above the flow parameter k are integrated out. This follows closely Wilson's idea of integrating out modes by momentum shells.

   In class we considered how the free energy $F(\psi)$ rescaled as the log of the cutoff length ℓ.

   On the left-hand side of eqn (28) $\partial_t \Gamma_k[\phi]$, what is the analogue of ℓ? What is the analogue of ψ? What functional encapsulates the information that we stored in the free energy F and how is it different to it? Can you think of a thermodynmic potential that is even more similar to the effective action? On the right hand side, is R_k somehow related to the idea of integrating out momentum shells?

(Submit electronically to Florian Theuss (ft226@cornell.edu) by 9:30 Monday evening; cc Sethna.)

Exercises

1. **Coarsening: Non-thermal fixed point..**[3] ③

   Read: Section 1 of [2].

   Another way to understand coarsening in isolated system is through a non-thermal fixed point[2], where the evolution after a quench is postulated to follow a self similar

[2]Problem developed by Florian Theuss, 2017.
[3]Problem developed by Hil Fung Harry Cheung, 2017.

evolution, with mode occupancies $f(\vec{k}, t) = t^\alpha f_S(\xi \equiv t^\beta |\vec{k}|)$. Note here we replace momentum p with wave vector k.

The coarsening scaling hypothesis can be expressed as the equal time correlator at time t has the form $C(\vec{r}, t) = f(|\vec{r}|/L(t))$. The structure factor $S(\vec{k}, t)$ is the Fourier transform of $C(\vec{r}, t)$, i.e. $S(\vec{k}, t) = \int d^d r C(\vec{r}, t) e^{i\vec{k}\cdot\vec{r}}$

Note - this self similar evolution has different scaling form at low k and high k. For studying coarsening, we will focus on the long distance (low k) scaling from.

Note 2 - the simulation is done for $d = 3$, where the analytics should be valid for general d.

Show that $S(\vec{k}, t) = L(t)^d g(|\vec{k}|L(t))$ for some universal function g.

Assume the system follows coarsening dynamics and assume mode occupancies $f(\vec{k}, t) = t^\alpha f_S(\xi \equiv t^\beta |\vec{k}|)$ equals structure factor $S(\vec{k}, t) = L(t)^d g(|\vec{k}|L(t))$ with $L(t) \propto t^{1/z}$. Express α, β in terms of dynamic exponent z, and dimension d.

From α, β in equation (2) of [2]) ($\alpha = \beta d, \beta = 1/2$), what is the corresponding z? Does this z depend on dimension d?

2. Cardy, exercise 4.4 (Crossover in an Ising slab.)

References

[1] Hohenberg, P. C. and Halperin, B. I. (1977, Jul). Theory of dynamic critical phenomena. *Rev. Mod. Phys.*, **49**, 435–479.

[2] Piñeiro Orioli, Asier, Boguslavski, Kirill, and Berges, Jürgen (2015, Jul). Universal self-similar dynamics of relativistic and nonrelativistic field theories near nonthermal fixed points. *Phys. Rev. D*, **92**, 025041.

[3] Williamson, Lewis A. and Blakie, P. B. (2017). Coarsening dynamics of an isotropic ferromagnetic superfluid.