Physics 7653: Statistical Physics
http://www.physics.cornell.edu/sethna/teaching/653/
Material for Week 12
Exercises due Tuesday Nov. 21
Last correction at November 14, 2017, 7:42 pm

Pre-class Preparation

Thursday

Read: Are randomly grown graphs really random?, Duncan S. Callaway, John E. Hopcroft, Jon M. Kleinberg, M. E. J. Newman, and Steven H. Strogatz, *Phys. Rev. E* **64**, 041902 (2001).

1. **Generating functions and grown networks.**[1] ⓟ

   Read "Are randomly grown graphs really random?", Duncan S. Callaway, John E. Hopcroft, Jon M. Kleinberg, M. E. J. Newman, and Steven H. Strogatz, *Phys. Rev. E* **64**, 041902 (2001).

   The network model described in the reading exhibits an infinite order phase transition reminiscent of the Kosterlitz-Thouless transition in the XY model. Near the critical point, the largest connected component size scales as $S(\delta) \sim e^{\alpha(\delta-\delta_c)^{-1/2}}$, where δ is the probability of adding an edge to the network at each time step and $\delta_c = 1/8$, is the point where the largest connected component size becomes non-zero.

   To find the critical point, we use the generating function

$$g(x) = \sum_{n=1}^{\infty} b_k x^k, \tag{1}$$

   where b_k is the probability of a randomly chosen vertex being in a (finite) component of size k. Then δ_c is the point where the average size of finite components $\langle s \rangle = g'(1)/g(1) = \sum_k k b_k / \sum_k b_k$ has a discontinuity.

   (a) *The second moment $\langle s^2 \rangle$ gives us information about how the component size varies in each realization of the grown network. Compute $\langle s^2 \rangle$ in terms of the generating function and its derivatives $g(1)$, $g'(1)$, and $g''(1)$. Using Eq. 11, evaluate $g''(1)/g(1)$ when $g(1) \neq 1$, is this scaling familiar? (Hint: $S = 1 - g(1)$.)*

   (b) *The authors claim that the difference between this network and the static network is the presence of degree correlations, measured by ρ (Fig. 5). Do you find this explanation satisfying? Do you think correlations explain the smoothness of the transition?*

[1]Problem developed by David Hathcock, 2017.

(Submit electronically to David Hathcock (dch69@case.edu) by 9:30 Wednesday evening; cc Sethna.)

Tuesday

Read: Conformal field theory and *c*-theorem, Qingyang Xu (CFTNotesQingyang.pdf distributed by e-mail).

(Qingyang Xu). Do Exercise 2.1 in Qingyang's notes.
(Submit electronically to Qingyang Xu (qx63@cornell.edu) by 9:30 Monday evening; cc Sethna.)

Exercises

- Exercise 2.2 in Qingyang's notes.
- Exercise 3.1.1 in Qingyang's notes.
- Also recommended: Exercises 3.3.1, 3.4.1, and 3.5.1, for building physical intuition.