

# Nonlinear Fits: Challenges and algorithms

Intro

I would like to apologize for missing this fantastic gathering of all my favorite colleagues. My wife is just now beginning to enjoy her new knee, but it seemed premature for me to go off to Europe without her. Jean-Philippe deserves applause for bringing this conference together, under such an inspiring rubric.

I did my PhD with Phil Anderson. He had an amazing track record of ideas that have shaped physics.

We are gathered here to explain why nature is comprehensible. As Eugene Wigner asked, why does mathematics describe the natural world? Phil Anderson asserted that there are emergent laws describing the collective behavior of complex systems — that the laws governing our economy or ecosystem are every bit as ‘fundamental’ as those of high-energy physics.

Today I want to introduce a different kind of emergence. We shall use information geometry and a kind of interpolation theory to show that complex systems with many microscopic parameters will quite generally exhibit comprehensible collective behaviors.

# *Gets stuck at bad fit*

Usually not lots of local minima!

Sloppy directions co-dimension argument

Robustness to external conditions

Lost on plateaus

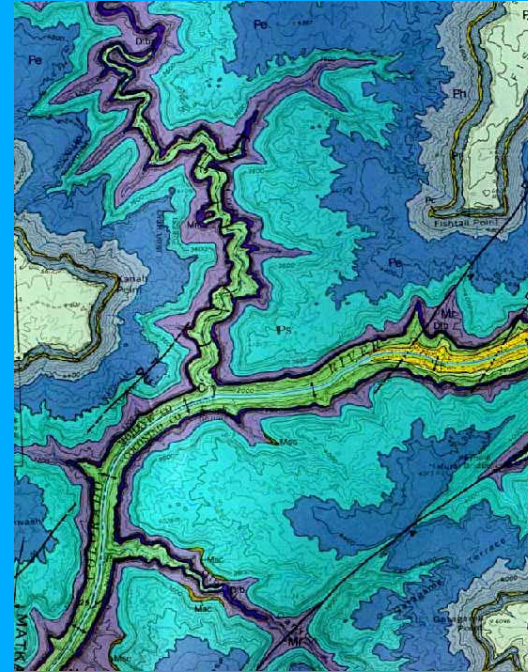
Twisty rivers at bottom

If most of the 48 dimensions of parameter space are sloppy, why not develop **simpler models** with fewer parameters?

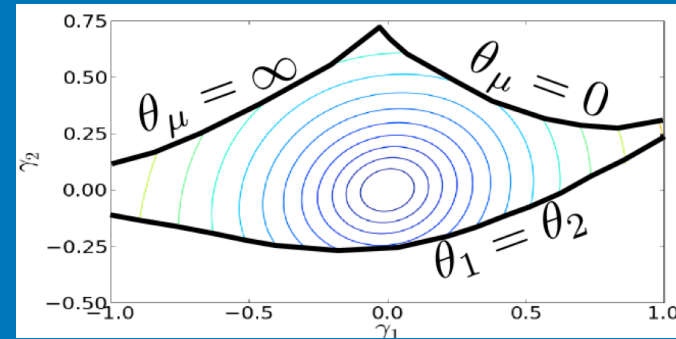
# Exploring Parameter Space

Rugged? More like Grand Canyon (Josh)

Glasses: Rugged Landscape  
Metastable Local Valleys  
Transition State Passes  
Optimization Hell: Golf Course  
Sloppy Models  
Minima: 5 stiff, N-5 sloppy  
Search: Flat planes with cliffs



## Finding best fit not hard in behavior space



The cost contours on the model manifold are concentric spheres near the best fit.

### Cost Contours in Behavior Space

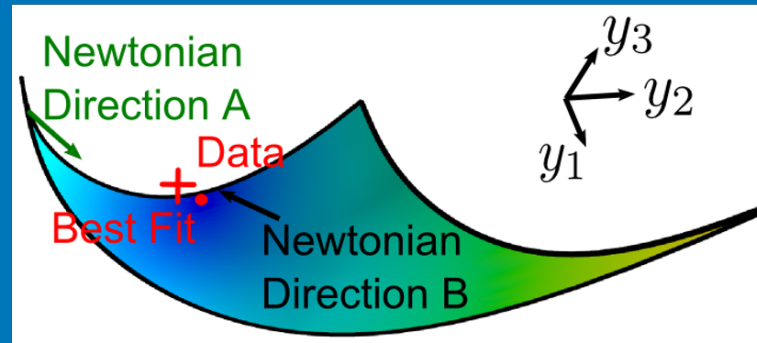
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# Gradient and "Newtonian" Directions



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**Gradient**  
= Downhill in *parameter* space: steepest descents

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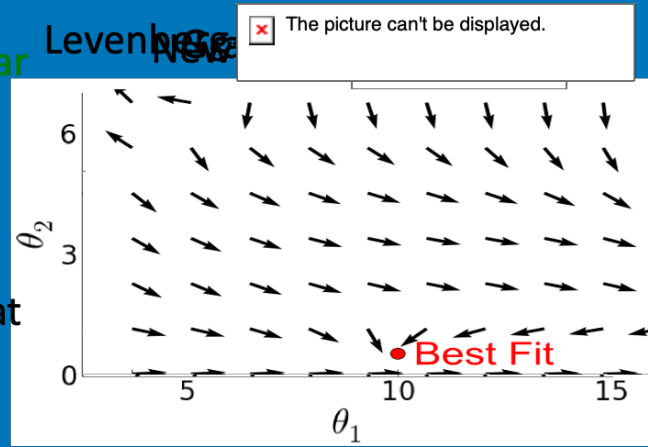
**Newtonian**  
= Downhill in *data* space

Newtonian can "evaporate" near boundary of model manifold.

Levenberg metric

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compromise; puts boundaries at infinity; transition to steepest descent

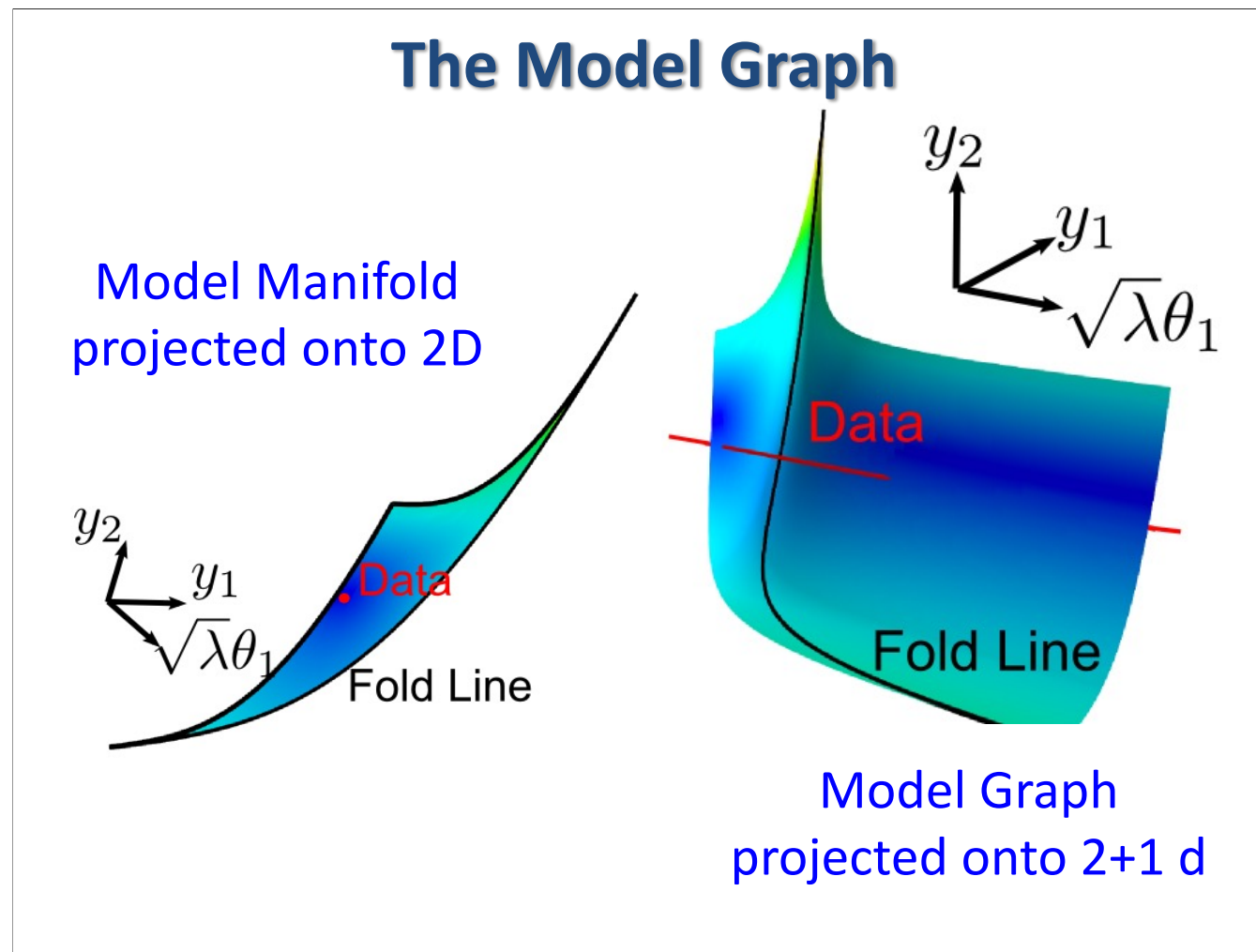


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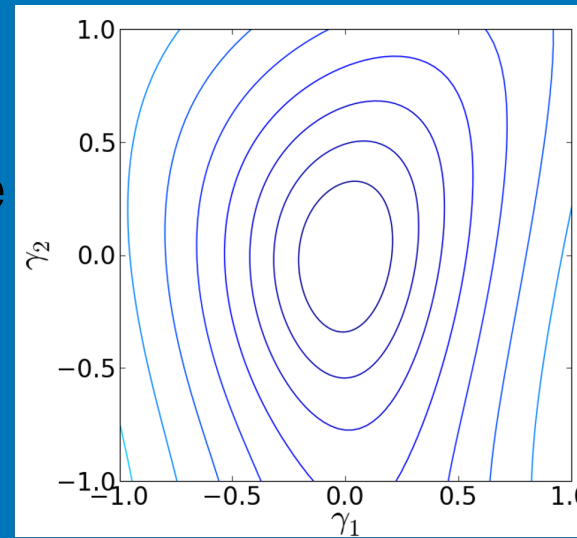
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## Finding best fit not hard in behavior space

The cost contours on the model graph are concentric spheres for small  $\lambda$ , parameter-space contours for large  $\lambda$ . (Here shifted to log parameters, so sloppiness and plateau not as apparent.)



Cost Contours in Behavior Space

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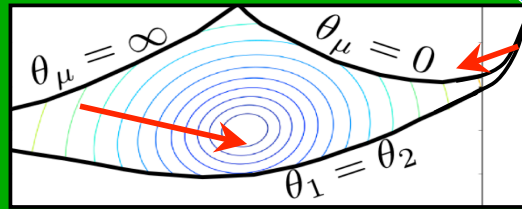
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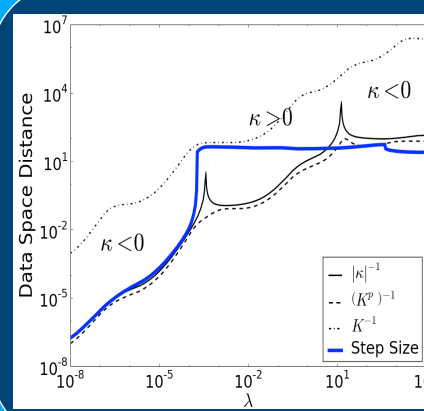
## B. Finding best fits: Geodesic acceleration



Geodesic Paths nearly circles  
Follow local geodesic velocity?

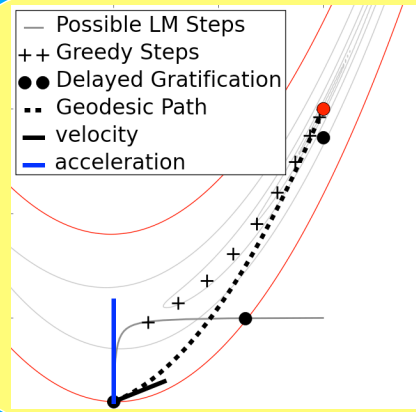
$$\delta\theta^\mu = -g_{\mu\nu} \nabla_\nu C$$

- Gauss-Newton
- Hits manifold boundary



### Model Graph

add weight  $\lambda$  of parameter metric yields Levenberg-Marquardt: Step size now limited by curvature

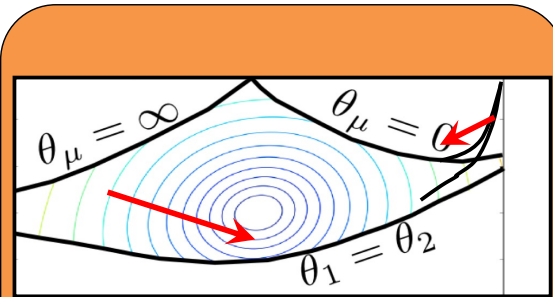


Algorithm	Success Rate	Mean njev	Mean nfev
Traditional LM + accel	65%	258	1494
Traditional LM	33%	2002	4003
Trust Region LM	12%	1517	1649
BFGS	8%	5363	5365

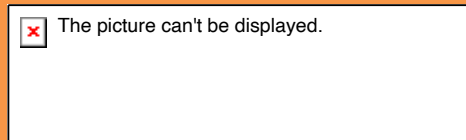
Follow parabola, **geodesic acceleration**  
Cheap to calculate; faster; more success



# Finding best fits: Geodesic acceleration

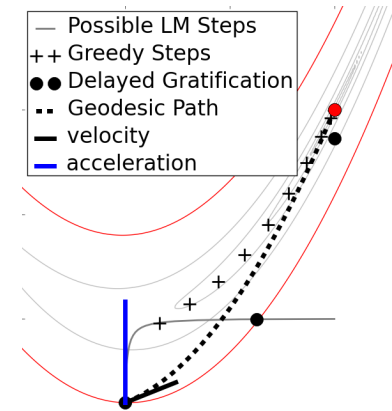


Geodesic Paths nearly circles (small  $\lambda$ )  
Follow local geodesic downhill?



**Geodesic acceleration**  

$$a^\mu = -\Gamma^\mu_{\alpha\beta} v^\alpha v^\beta$$
 (quadratic approximation to geodesic equation)



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Follow parabola, **geodesic acceleration**  
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If most of the 48 dimensions of parameter space are sloppy, why not develop **simpler models** with fewer parameters?

***LM = Downhill on MG  
Yellow sticky section  
Delayed gratification  
Geodesic Acceleration  
(Not much faster, but  
doesn't get stuck as  
often)***

If most of the 48 dimensions of parameter space are sloppy, why not develop **simpler models** with fewer parameters?

# Applications not discussed here

Dynamical systems:

Lyapunov and structural susceptibility (Chachra, Transtrum)

Power system dynamics (Transtrum)

Optimal experimental design (Casey, Cerione; Tidor et al.)

Control theory

Nuclear physics

Protein allostery

Heart dynamics