Why does science work? Information geometry, multiparameter models, and the emergence of simplicity

Module #1, Physics 7654, Basic Training in Condensed Matter Physics

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Syllabus

Complex models in physics, biology, economics, and engineering are usually successful because their microscopic details are not crucial for describing the real world. We explain their success using information geometry, combining differential geometry with approximation theory to understand the reason simplicity emerges from complex underpinnings.

My lecture slides with notes, reading materials, and exercises will be available on Canvas (see the Modules), and perhaps will be posted on the Basic Training Web site (if accessing Canvas is a problem). I will attempt to tape my lectures as well, and make the tapes accessible on Canvas. If that is successful, I'll ask your permission to make them accessible on the Web (including questions and discussions that you may have participated in). Permission is completely voluntary, and I will ask for anonymous objections before passing out forms.

A rough sequential outline of the material to be covered in the four weeks of this module:

1. Emergence without small parameters: Sloppy models.
    - Emergence with small parameters: Continuum limits and renormalization
    - Emergence without small parameters: Engineering, biology, finance, and much of physics
    - Systems biology is sloppy
    - Eigenvalue hierarchies are ubiquitous
    - Litany of applications
        - CESR optimization, interatomic potentials,
        - cosmic microwave background radiation
        - deep neural networks, macroeconomics,
        - climate modeling
2. Sloppy models and parameter space
    - Predictions without known parameters
    - Sloppiness in systems biology
    - Hessians and Jacobians, Fisher Information Metric, deep connections to statistical mechanics
    - Sloppy spectra and the renormalization group
    - Applications
        - Knowing parameters is useless.
        - Sloppiness vs. robustness in systems biology.
        - Evolutionary biology, sloppiness, and neutral spaces.

    - Estimating *systematic* errors in atomistic simulations and electronic structure theory.

3. Model manifolds in behavior space: Hyperribbons
    - Model manifolds, metrics, geodesics
    - Low-dimensional hyperribbon behavior manifests sloppy parameters
    - Hyperribbons are ubiquitous
    - Hyperribbons as emergent simplicity
    - Curvatures: intrinsic, extrinsic, and parameter effects curvature
    - Rigorous hyperellipsoid bounds on model manifold: sloppy polynomials
4. Challenges in multiparameter fits to data
    - Sloppy models not rugged
        - minima easy to find in behavior space
        - infinite plateaus and windy canyons in parameter space
    - Parameter evaporation to plateaus at edges of model manifold
    - Geometric interpretation of least-squares fit algorithms as motion on model *graph*
    - Delay gratification to avoid traps deep in canyon basins
    - Geodesic accelleration to move bigger steps along canyons
5. Emergent simplicity
    - Hyperribbons, intuition, wisdom, experience, and superstition
    - Hyperribbons as stepping stones from spherical cows to refined understanding
    - Pruning using geodesics to the model boundary: MBAM
    - Bayesian priors that work: emphasizing the model boundaries
        - Jeffrey's prior is evil
        - Occam's razor and model boundaries
        - Slab-and-spike prior
        - Mutual information prior
6. Visualizing model behavior in probability space
    - Visualizing nonlinear least-squares models: PCA and SVD
    - Probabilistic models and a curse of dimensionality
    - Probability space is a sphere!
    - Hellinger embedding is doomed
    - Taking the limit of zero data: replica theory and intensive PCA
    - isKLe, statistical mechanics, and the Kullback–Leibler divergence
    - Applications
        - the Ising model
        - deep neural networks,
        - the manifold of possible Universes