Group project: preparation for computer lab I.
Computational Physics 4480/7680, Astro 7690

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Before coming to computer lab Friday Jan. 31, please prepare answers to the following three warmup questions:

1. **Timing Sine** (from problem set 1): *Build a table of a million equally-spaced numbers* $0 \leq x_n < 2\pi$. *Find a method, inside your working environment, for calculating the amount of time a computation takes. Time how long it takes to calculate* $\sin(x_n)$ *and* x_n^2 *for your million points.*

2. *What is the Taylor series approximating*

$$\sin(x) = \sum_{n=0}^{N-1} a_n x^n \tag{1}$$

   about $x = 0$*? If we assume the error in the truncated series is roughly given by the first neglected term* $a_n x^n$*, how big must* N *be before the absolute error for* $\sin(2\pi)$ *is less than double-precision machine accuracy* $\epsilon_m = 2.22 \times 10^{-16}$*? Can we hope to reduce the* fractional *error to below* 1% *at* $x = 2\pi$*?*

3. *What is the maximum value of the second derivative* $\sin''(x)$*? If we expand*

$$\sin(x) \approx \sin(n\Delta) + (x - n\Delta)\sin'(n\Delta) + (x - n\Delta)^2/2!\sin''(n\Delta), \tag{2}$$

   how small must Δ *be for the linear approximation error to be less than* ϵ_m*?* (Hint: the maximum distance to the nearest data point is $\Delta/2$.)