Harmonic Oscillators: Raising and lowering operators

Harmonic oscillator Hamiltonian in one dimension. Ground state ψ_0. Check normalization. Plot ground state, evaluated at ...

Hints: The shortcut for subscripts is "Ctrl -", and superscripts is "Ctrl ^". The shortcut for $\hbar$ is "esc hb esc", ∞ is "esc inf esc", π is "esc p esc", and ω is "esc w esc".

Define ψ_0 symbolically. Is it normalized?

```
ψ0 := (m ω / (π ℏ)) ...;
Integrate[ ..., {x, -∞, ∞}, Assumptions → {m > 0, ω > 0, ℏ > 0}]
```

We take our constants from McEuen's bouncing buckyballs, Park et al., "Nanomechanical oscillations in a single-C_{60} transistor", Nature 407, 57 (2000).

```
amu = 1.660538782 × 10^-24;
constants = {m →  ..., ℏ → 1.054571628251774 × 10^-27,  ...};
a0 =  ... /. constants;(* RMS width of Gaussian *)
ψ0 /. constants // Simplify

Plot[ψ0 /. constants, {x,  ...,  ...}, PlotRange → All]
```

How do the zero-point fluctuations for McEuen' s bouncing buckyball compare to the size of an atom?

Note the Hamiltonian is an operator mapping functions into functions. H[ψ] is a new function. We define a function on function space that returns H[ψ]. You can take the second derivative of ψ as D[ψ, {x, 2}].

```
H[ψ_] :=  ...

H[ψ0] / ψ0 // Simplify
```

Momentum operator p. Creation operator a^+. Exciting the ground state using creation operators.

```
p[ψ_] := ...
a+[ψ_] :=  ...

H[a+[ψ0]] /. .. // Simplify

Integrate[ ...]

a+[a+[ψ0]] // Simplify
```

```
Integrate[ ...]
```

This isn't the eigenstate ψ_2, since it has the wrong norm. Try taking the norm of higher powers of `a⁺[a⁺ ...[ψ0]] ...` until you figure out what to divide by to normalize it.

```
Integrate[ ...]
```

Defining normalized eigenstates. Plots.

Hint: We are using recursion, defining ψ_n in terms of ψ_{n-1}. It should be $a^{+}\psi_{n-1}$ up to a constant that depends on n. You can either figure this out analytically using commutation relations, or by working by analogy from the norms above...

```
ψn_ := ... a⁺[ψn-1] // Simplify

Integrate[ψ4^2, ...]

{ψ0, ψ1, ψ2, ψ3, ψ4}

Wavefunctions = {ψ0 /. constants, ψ1 /. constants, ψ2 /. constants, ψ3 /. constants}

Plot[Wavefunctions, {x, ...}, PlotRange → All]
```