Problem Set 1: Notation
Graduate Quantum I
Physics 6572

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Due Friday Aug 29 (day after tomorrow!)
Last correction at August 20, 2014, 8:47 pm

Potentially useful reading

Sakurai and Napolitano, sections 1.1 (Stern-Gerlach), 1.2-1.3 (Notation)
Weinberg, sections 3.1-3.3 (Notation)
Schumacher & Westmoreland chapter 2 (Stern-Gerlach, qbits)
Summary of Quantum Mechanics, pp. 1-14 from
The Quantum Mechanics Solver, Jean-Louise Basdevant and Jean Dalibard

1.1 **Four slits** (Schumacher & Westmoreland problem 1.6). A single photon passes through a barrier with four slits and strikes a screen some distance away. Consider a point X on the screen. The probability amplitudes for reaching X via the four slits are ϕ_1, ϕ_2, ϕ_3, and ϕ_4.

(a) *What is the net probability P that the photon is found at X if no measurement is made of which slit the photon passed through?*

(b) *A detector is placed by slit #4, which can register whether or not the photon passes through the slit (but does not absorb the photon or deflect it). What is P in this case?*

(b) *The detector is now moved to a point between slits #3 and #4 and registers whether or not the photon passes through one of these slits. However, the detector does not record which of these two slits the photon passes. What is P in this case?*

1.2 **Quantum Notation.** (Notation) ②

For each entry in the first column, match the corresponding entries in the second and third column.

Schrödinger	*Dirac*	*Physics*
$\psi(x)$	*A.* $\langle\psi\|x\rangle p/2m\langle x\|\psi\rangle - \langle x\|\psi\rangle p/2m\langle\psi\|x\rangle$	*I. The number one*
$\psi^*(x)$	*B. Ket* $\|\psi\rangle$ *in position basis, or* $\langle x\|\psi\rangle$	*II. Probability density at* x
$\|\psi(x)\|^2$	*C. Bra* $\langle\psi\|$ *in position basis, or* $\langle\psi\|x\rangle$	*III. Probability* ψ *is in state* ϕ
$\int \phi^*(x)\psi(x)\mathrm{d}x$	*D.* $\langle\psi\|x\rangle\langle x\|\psi\rangle$	*IV. Amplitude of* ψ *at* x
$\|\int \phi^*(x)\psi(x)\mathrm{d}x\|^2$	*E. Braket* $\langle\phi\|\psi\rangle$	*V. Amplitude of* ψ *in state* ϕ
$\int \|\psi(x)\|^2\mathrm{d}x$	*F.* $\langle\psi\|\psi\rangle$	*VI. Current density*
$(\hbar/2m\mathrm{i})(\psi^*\nabla\psi - \psi\nabla\psi^*)$	*G.* $\langle\psi\|\phi\rangle\langle\phi\|\psi\rangle$	*VII. None of these.*

There will be a computational exercise almost every week, typically with hints files. Mathematica, Python, and Octave are available in the Rock B3 computer lab. Python and Octave are freely downloadable; Octave is the freeware version of Matlab. Mathematica is a useful tool available for a reasonable price from the campus store: see http://www.it.cornell.edu/services/software_licensing/available/mathematica.cfm.

1.3 **Square well ground state.**[1] (Quantum numeric) ②

Consider a particle of mass M in one dimension, confined in a potential that vanishes for $-a \leq x \leq a$ and becomes infinite at $x = \pm a$, so the wavefunction must vanish at $x = \pm a$.

Find the normalized ground state $\psi_0(x)$*. Suppose that the particle is placed in a state* ψ *with a wave function proportional to* $a^2 - x^2$*, which vanished properly at the edges. If the energy of the particle is measured, what is the probability that the particle will be found in the state of lowest energy?* Use a computational method (symbolic or numerical) to avoid tedious evaluation of integrals. *Evaluate your answer numerically (it should not depend on* a*). Also, plot the ground state and the trial state, using* $a = 3$*; they should look rather similar.* This exercise may take much longer than it seems, since you'll likely spend some time setting up your computational environment – installing software, getting plots to work, and so forth. The Hints files should help once the software is set up.

[1] Adapted from Weinberg, *Lectures on Quantum Mechanics*, ex. 1.1. Hints files available on the course Web site for Mathematica, Python, and Octave/Matlab).