Group Project: Computational Path Integrals
Graduate Quantum I
Physics 6572

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Due Friday Nov. 16

Last correction at October 29, 2012, 10:28 am

Reading

Sydney Coleman, "The Uses of Instantons", in *Proc. Int. School of Subnuclear Physics*, (Erice, 1977); and in *Aspects of Symmetry* p. 265, Cambridge University Press, 1985; Feynman and Hibbs, *Quantum Mechanics and Path Integrals*
Decay Rates of Tunneling Centers Coupled to Phonons: An Instanton Approach, James P. Sethna, Phys. Rev. B 25, 5050 (1982)
"Phonon Coupling in Tunneling Systems at Zero Temperature: An Instanton Approach, James P. Sethna, Phys. Rev. B 24, 698 (1981).

Each group should generate a numerical exercise, suitable for distribution in following years of this course, utilizing and illustrating the path-integral approach to quantum mechanics. The goal is to reach the largest amount of conceptual understanding with the smallest investment of effort and technical glitches.

I have several thoughts on possible exercises, listed below. My guess is that if I were to pursue these ideas, about half could make for good exercises, about a third would turn out to be either numerically infeasible or conceptually confusing, and the rest wouldn't even make sense once I got into them. I'll be interested to see how they work for you, but I'll be happy with intelligent effort and thought even if not everything works.

In rough order of ambition and likelihood of success

1. Discrete path integral for two-level system. The Hamiltonian for a two-level system $\mathcal{H} = \left(\begin{smallmatrix} -\epsilon & -\Delta \\ -\Delta & \epsilon \end{smallmatrix}\right)$, can represent an atom tunneling in a solid, a spin 1/2 particle, or a number of other systems. Solve analytically for the propagator. Calculate the propagator as a path integral, where the paths 'flip' between up and down states, summing over the times of the flips. This exercise could focus on the derivation of the path integral through taking small time steps.

2. Prefactor for WKB tunneling rate in quartic double well. Numerically calculate the integral over quadratic fluctuations that Coleman calculates analytically using Wronskians.

3. Two-level system coupled to a single harmonic oscillator. This can be solved analytically in the 'rotating wave' approximation, I suspect; Sakurai has a section on it. There are two methods for doing the path integral: explicitly doing a path integral over the continuous harmonic oscillator and the discrete two-state system, or integrating out the harmonic oscillator using Feynman's action, giving an interaction between 'flips'.

4. Double-slit experiment. I'm not sure how to set this up, but a path integral for double slits would be great.

5. Real-time path integral for double well tunneling rate. The idea is to do a brute-force integral over intermediate times, and see oscillations. The key is to add some kind of regularization, so that the paths with large excursions and fast wiggles get suppressed in the integrals. I'm guessing a slight rotation into imaginary time or something might work.

6. Two level system, or even a double well, coupled to a heat bath of harmonic oscillators. This is roughly what I did my thesis on. One can integrate out the harmonic oscillators, get a time-retarded interaction, and I think do the time-retarded stuff by Fourier transforms. Decay rates, effective masses, overlap integrals, etc.