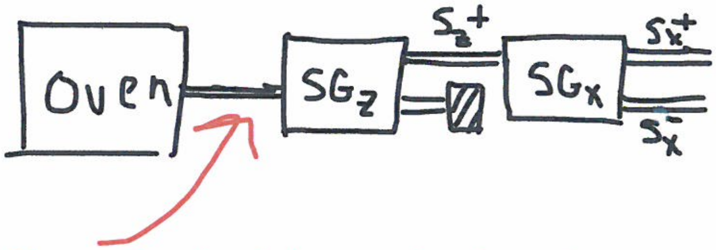
Entanglement



State of Stern Gerlach beam before first split? Unpolarized

~~Not~~ $\underbrace{\frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle}_{\text{Quantum Superposition}}$! What is it? $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

Want Classical mixture "half" $|\uparrow_z\rangle$ & "half" $|\downarrow_z\rangle$

Leads us into entanglement (today) and density matrices (later)...

How is the atom beam created? Thought experiment.

Diatomic molecule (Ag_2) w/ atoms (Ag) w/ L=0, I=0, and S=1/2

~~(None exist: I checked.)~~

Ground state spin singlet.

Spin state $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R\right)$

* Spin wavefunction (WF) antisymmetric: L -> R gives -1.
* Electrons are fermions.
* Net electron WF antisymmetric (nuclei = potential)
* Ignoring spin-orbit coupling (much later), WF factors

$$\Psi(x_R, s_R, x_L, s_L) = \chi(s_R, s_L)\,\psi(x_R, x_L) = \left(\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\right) \text{(6 Dim)}$$

so if $\chi(s_R, s_L) = -\chi(s_L, s_R)$, then $\psi(x_R, x_L) = \psi(x_L, x_R)$

He noninteracting spin

groundstate Fermi Exclusion $\Rightarrow$ $1s^2$ $\uparrow\downarrow$

Now $1s^2\left(\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\right)$!

Model oven splitting molecule, not altering spins

$\Psi = ($  $) \times \left((|\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R \right) / \sqrt{2}$

Behind $\quad \vec{B}$eam $\quad$ 50% $\uparrow, \downarrow$

Two quantum systems are 'entangled' if

* $|\psi_{AB}\rangle$ cannot be written as $|\psi_A\rangle |\psi_B\rangle$

Spin WFs of L & R are entangled. But how do we tell?

$|\uparrow_L\rangle |\downarrow\rangle_R$ entangled? No.

$\frac{1}{2}(|\uparrow\rangle_L |\uparrow_R\rangle + |\uparrow_L\rangle |\downarrow_R\rangle + |\downarrow\rangle_L |\uparrow\rangle_R + |\downarrow\rangle_L |\downarrow\rangle_R)$ entangled?

No: equals $|\uparrow_x\rangle_L |\uparrow_x\rangle_R = (|\uparrow_z\rangle_L + |\downarrow_z\rangle_L)(|\uparrow_z\rangle_R + |\downarrow_z\rangle_R)/2$

How do we tell? General decomposition: $|\varphi_\alpha\rangle$ complete in A, $|\chi_\beta\rangle$ complete in B,

~~General~~ $\quad |\psi_{AB}\rangle = \sum M_{\alpha\beta} |\varphi_\alpha\rangle_A |\chi_\beta\rangle_B$

Schmidt Decomposition = Singular Value Decomp (SVD)

Check: Complex M SVD

Schmidt: $|\psi_{AB}\rangle = \sum_k \underbrace{\sigma_k}_{\text{positive}} \underbrace{|\varphi_A\rangle_k}\, \underbrace{|\varphi_B\rangle_k}$

orthonormal in A, B, Chosen for ψ_{AB}

complex $\; U \Sigma V^\dagger$

SVD: $\underbrace{M}_{\text{Any Matrix}} = U \underbrace{\Sigma}_{\text{Diagonal } \sigma_{11} > \sigma_{22} > \ldots >} V^T$ } Columns of U, V orthonormal

$$M_{ij} = \sum_k U_{ik} \Sigma_{kk} V_{jk}$$
$$= \sum \sigma_{kk} \vec{u}^{(k)} \otimes \vec{v}^{(k)}$$

Diagonalize any matrix via two rotations (domain, range)

σ_{kk} unique: unless $\sigma_{11} = 1, \sigma_{22} = \ldots = 0$, entangled

(Singular vectors almost unique, up to sign, degeneracy)

Is the singlet state entangled? Schmidt decomposition?

$\{|\uparrow\rangle_L, |\downarrow\rangle_L\}$ orthonormal in L

$\{-|\uparrow\rangle_R, |\downarrow\rangle_R\}$ orthonormal in R

$$\text{Singlet} = \frac{|\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R}{\sqrt{2}} = \underbrace{\frac{1}{\sqrt{2}}}_{\sigma_{11}} |\uparrow_L\rangle |\downarrow\rangle_R + \underbrace{\frac{1}{\sqrt{2}}}_{\sigma_{22}} |\downarrow\rangle_L (-|\uparrow\rangle_R)$$

Since SVD / Schmidt decomposition unique, singlet entangled

Einstein: Spooky action at a distance

What happens when S-G measures spin SzR of R atom?

$$\chi = \left(\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\right) = \left(\frac{\uparrow_L}{\sqrt{2}}\right)\downarrow_R + \left(\frac{\downarrow_L}{\sqrt{2}}\right)\uparrow_R \Rightarrow \begin{cases} \downarrow_L \uparrow_R & \text{prob} \left(-\frac{1}{\sqrt{2}}\right)^2 \\ \uparrow_L \downarrow_R & \text{prob} \left(\frac{1}{\sqrt{2}}\right)^2 \end{cases}$$

Instantly, left atom has definite S_z^L!

Copenhagen: Wavefunction 'collapse' (above)
Many worlds: Universe splits

$$\frac{1}{\sqrt{2}} |\uparrow_L \downarrow_R\rangle \left| \begin{matrix}\text{Screen}\\ \text{down dot}\end{matrix} \right\rangle - \frac{1}{\sqrt{2}} |\downarrow_L \uparrow_R\rangle \left| \begin{matrix}\text{Screen}\\ \text{up dot}\end{matrix} \right\rangle$$

Mermin 'Ithaca' interpretation: Quantum theory tells correlations: If R measures up, then L will measure down, and vice-versa

S-G Beam is R's leaving L's behind!

R-beam forgetting L 'half' up and 'half' down (density matrix, later)

Entropy = Quantification of amount of ignorance (general)

$$S = -k_B \sum P_i \log P_i \quad \text{for } P_i = \left\{\begin{matrix}\text{classical}\\ \text{probability}\\ \text{state } i\end{matrix}\right\} = \sigma_{ii}^2$$

Entanglement entropy = Entropy of R if L is forgotten

$$S = -k_B \sum_i \sigma_i^2 \log \sigma_i^2 = -\frac{k_B}{2} \log \frac{1}{2} - \frac{k_B}{2} \log \frac{1}{2}$$

$$= k_B \log 2 = k_B \log(\#\text{ of equally likely states})$$

Ignorance

Note: $\sigma_{ii} \rightarrow \sigma_i$ (Convention)

$\log \equiv \log_e = \ln$ (10 isn't important to physics)

k_B = Boltzmann's constant

sometimes $\rightarrow k_B = 1/\log 2$ so

entropy measured in <u>bits</u>

Sometimes $k = 1$, entropy in 'nats'

Entanglement entropy quantifies information lost when state of L is forgotten (coherence disrupted)...