Quantum Weirdness:
Bell's Theorem & GHZ

Einstein was bothered by the 'wave function collapse':

$$\underset{AB}{|\uparrow\downarrow\rangle} - \underset{AB}{|\downarrow\uparrow\rangle} \quad + \text{ A measures } \uparrow \;\Rightarrow\; \text{B must measure } \downarrow.$$

Perhaps ~~A spin~~ the spins have some complicated internal state (hidden variables) that makes this not so weird.

Bell: Hidden Variables Assume each spin has a definite answer to any measurement by A or B.

Locality Assume A's measurements do not affect B's, or vice versa.

(Locality is true in quantum mechanics!)

~~Then~~ Quantum mechanics violates these two hypotheses.

<u>Three</u> observers A, B, & C

Entangled state $|GHZ\rangle = \frac{1}{\sqrt{2}}\left(\cancel{|000\rangle + |111\rangle}\ |111\rangle - |000\rangle \right)$

Q: What is the probability, ~~that~~ if A B & C each measure X ~~$= \frac{1}{\sqrt{2}}(\ldots)$~~, that all three will measure $X = +1$?

Remember, $|1_z\rangle = \frac{1}{\sqrt{2}}\left(|1_x\rangle + |{-0_x}\rangle\right)$

$|0_z\rangle = \frac{1}{\sqrt{2}}\left(|1_x\rangle - |0_x\rangle\right)$

$Z|1_z\rangle = |1_z\rangle \quad Z|0_z\rangle = -|0_z\rangle$, same for X, Y

A: $|111\rangle = \frac{1}{2\sqrt{2}}\Big[(1_{xA} + 0_{xa})(1_{xB} + 0_{xB})(1_{xC} + 0_{xC})$

$|000\rangle = \frac{1}{2\sqrt{2}}(1_{xA} - 0_{xA})(1_{xB} - 0_{xB})(1_{xC} - 0_{xC})\Big]$

$$|GHZ\rangle = \frac{1}{4}\Big(0\overbrace{|111\rangle}^{XXX=1} + 2\cdot\overbrace{|000\rangle}^{XXX=-1}$$
$$+ 2\overbrace{(|110\rangle + |101\rangle + |110\rangle}^{XXX=-1}$$
$$+ 0(\underbrace{|100\rangle + |010\rangle + |001\rangle}_{XXX=+1})\Big)$$

$X_A X_B X_C |GHZ\rangle = -|GHZ\rangle$

Q: If Bell is right, can it be possible that $XYY\,|GHZ\rangle = +|GHZ\rangle$?

(By symmetry, $YXY\,|GHZ\rangle = +|GHZ\rangle$
and $YYX\,|GHZ\rangle = +|GHZ\rangle$

A: Surely, if X_A, Y_A, X_B, Y_B, and X_C, Y_C are all defined before the measurement, then ~~the value~~ since $Y_A^2 = 1$ whichever ± 1 value Y_A has,

$$(X_A Y_B Y_C)(Y_A X_B Y_C)(Y_A Y_B X_C) = X_A X_B X_C$$

But we saw $X_A X_B X_C = -1$, so XYY can't (if Bell is right) be an eigenstate of $+1$.

Q: Is $XYY\,|GHZ\rangle = +|GHZ\rangle$?

Remember $|1_z\rangle = \frac{1}{\sqrt{2}}(|1_y\rangle + |0_y\rangle)$

$$|0_z\rangle = \frac{i}{\sqrt{2}}(|1_y\rangle - |0_y\rangle)$$

A: $|1_z 1_z 1_z\rangle = \frac{1}{2\sqrt{2}}(1_x + 0_x)(1_y + 0_y)(1_y + 0_y)$

$|0_z 0_z 0_z\rangle = \frac{1}{2\sqrt{2}}(1_x - 0_x)\,i(1_y - 0_y)\,i(1_y - 0_y)$

$= -\frac{1}{2\sqrt{2}}(1-0)(1-0)(1-0)$ } minus previous calculation

$|GHZ\rangle = \frac{1}{\sqrt{2}}(|1_z 1_z 1_z\rangle - |0_z 0_z 0_z\rangle)$

$= \frac{1}{4}(\overbrace{2|111\rangle}^{XYY=+1} + \overbrace{2\{|100\rangle + |010\rangle + |001\rangle\}}^{XYY=+1})$ Weird