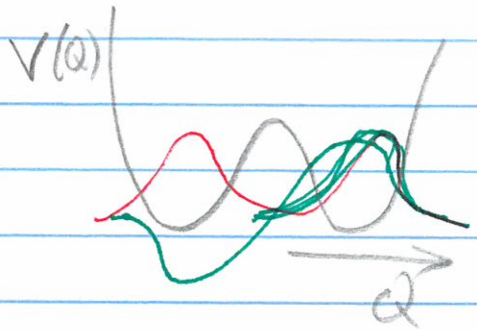
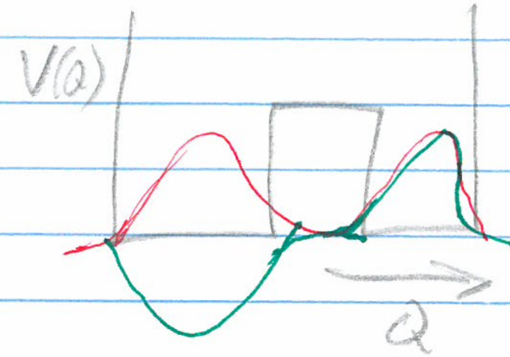
- No HW next ~~two~~ weeks (Prelim) ~~Prelim, then break~~)
- WKB/Instantons & Photon ~~Density~~ matrix exercises! "Flipped Classroom"

WKB Review





Symmetric Well

Tunnel splitting 2Δ

$\Downarrow$

Two-Level System $\begin{pmatrix} 0 & -\Delta \\ -\Delta & 0 \end{pmatrix}$

$\Delta \sim$ Overlap of 'left' + 'right' states.

$$\underbrace{\text{WKB} \wedge \Delta}_{\text{(Instantons)}} \sim \Delta_0 e^{-\int \sqrt{2mV(Q)}\, dQ / \hbar}$$

$$H\psi = E\psi$$

$$(H-E)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + (V-E)\psi = 0$$

$$\psi \sim e^{-kx} \qquad \frac{\hbar^2 k^2}{2m} = V - E$$

$$k \sim \sqrt{2mV}/\hbar$$

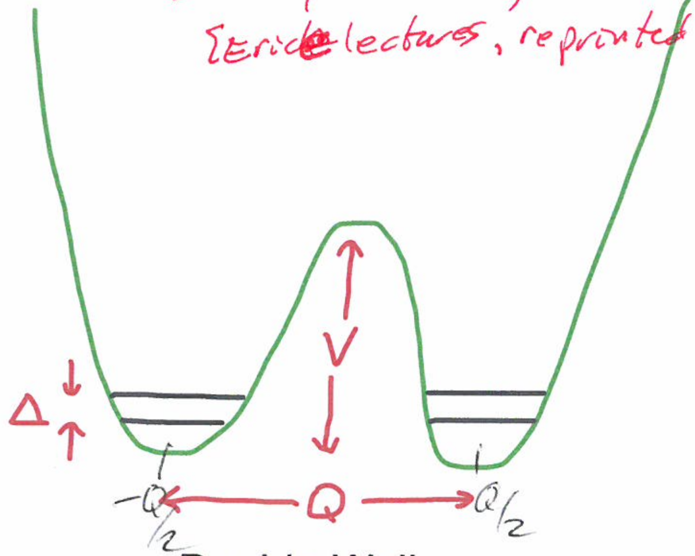
$$\boxed{\psi \sim e^{\pm\sqrt{2mV}\, x/\hbar}}$$

Overlap

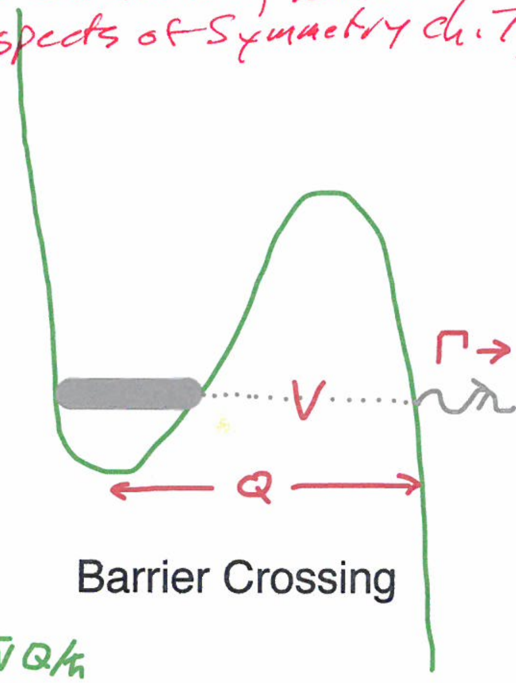
$$e^{-\sqrt{2mV}\,x} \qquad e^{+\sqrt{2mV}\,x}$$

Instantons, Quantum Tunneling, and WKB

Sidney Coleman, The Uses of Instantons, sections 1&2 [Erice lectures, reprinted in Aspects of Symmetry ch. 7.]



Double Well



Barrier Crossing

$$\text{WKB:}\ \Delta \sim e^{-\sqrt{2MV}\,Q/\hbar},$$

$$\Gamma \sim \Delta^2$$

How to use path integrals? Rotate to imaginary time!

$$\langle x', t' | x_0, t_0 \rangle = \iint \mathcal{D}[x(t)]\; e^{\frac{i}{\hbar}\int \frac{1}{2} m\dot{x}^2 - V(x)\, dt}$$

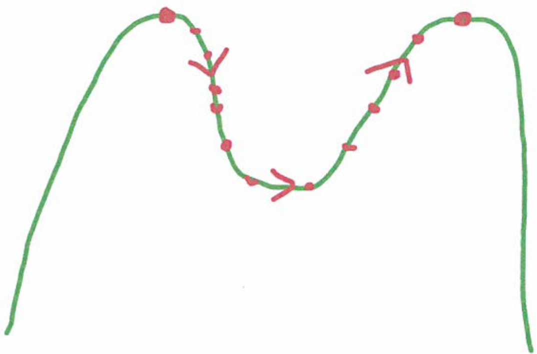
$$\tau = it; \quad -i\,d\tau = dt; \quad \dot{x}^2 = (dx/dt)^2 = -\left(\frac{dx}{d\tau}\right)^2 \equiv x'^2$$

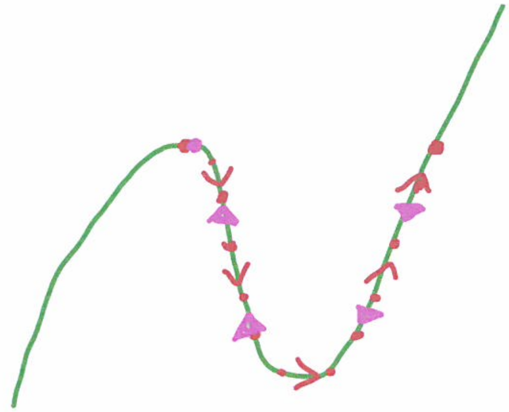
$$= \iint \mathcal{D}[x(\tau)]\; \exp\left[-\frac{1}{\hbar}\int \frac{1}{2} m x'^2 + V(x)\, d\tau\right]$$

* Analytic continuation???
* Physics, not math. Gives different information!
* No oscillations! Biggest minimizes Euclidean action:

$S_E = \int \frac{1}{2} m x'^2 + V(x)\, d\tau$ = <u>Inverted</u> potential

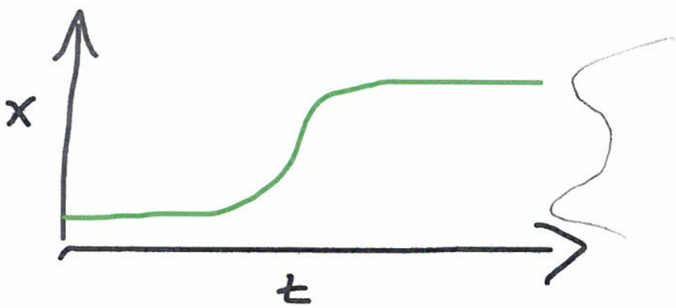
Assume $V(x) = 0$ at $x = \pm Q/2$

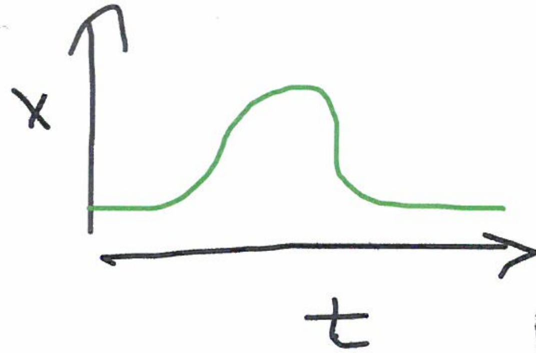




Instanton: Falls in, rolls up other side

Barrier crossing: bounces off turning point

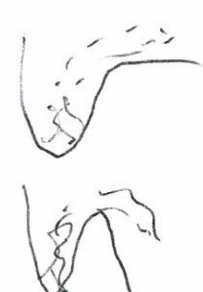




Soliton in time -> Instanton

Instanton Bounce
Crosses barrier twice

Decay = Destructive interference
Path must cross twice



• Dilute Gas

• Quadratic fluctuations

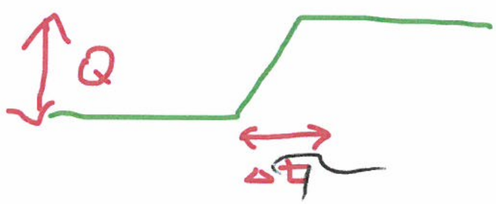
Stationary phase

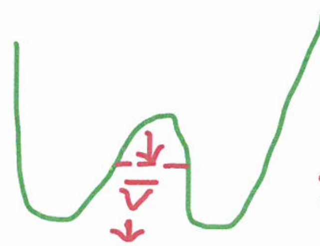
$$\underbrace{e^{iS_0/\epsilon}}_{\text{we're keeping}}\ \underbrace{\sqrt{\frac{2\pi\epsilon}{S''}}}_{\text{prefactor}}$$

$$\Gamma \sim \Delta^2$$

How to get WKB?

(1) Variational bound: Ramp





$$S_E \le \tfrac{1}{2} M\, Q^2/\Delta\tau^2\, \Delta\tau + \bar{V}\Delta\tau$$

Minimize wrt Δt: $-\tfrac{1}{2} M Q^2/\Delta\tau^2 + \bar{V} = 0$

$$\Delta\tau = \sqrt{\tfrac{1}{2} M Q^2/\bar{V}}$$

$$S_E \lesssim \sqrt{\tfrac{1}{2} M Q^2 \bar{V}} + \sqrt{\tfrac{1}{2} M Q^2 \bar{V}} = \sqrt{2M\bar{V}}\, Q$$

$$\Delta = \underbrace{\{\hbar\omega_0\}}_{\text{Fluctuations}} e^{-\sqrt{2M\bar{V}}\, Q/\hbar}$$

$$\bar{V} = \int V(x)\,\frac{dx}{Q}$$

$$\int V(x(\tau))\,d\tau \qquad x = \frac{Q\tau}{\Delta\tau}$$

$$\Delta\tau \frac{dx}{Q} = d\tau$$

$$= \int V(x)\,\frac{dx}{Q}\,\Delta\tau$$