Rotations form a $\underset{\text{of physical twistings}}{\text{group}}$ SO(3)

Group = Set w/ multiplication law $g_1 g_2$

- Identity $e = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$
- Inverse g^{-1}
- Associative

Rotations act on vector spaces
⇒ matrix representation

$\vec{V} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ points in 3D

$R(g)\,\vec{v}$ = Rotated vector

$R: g \to 3\times3$ matrices

$$R(\hat{z},\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{3 dim rep}$$

Spin ½ representation [of SU(2)]

$$\mathcal{D}(\hat{z},\varphi) = \begin{pmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{i\varphi/2} \end{pmatrix} \quad \text{2-dim rep}$$

Wave functions form a representation

Hilbert space $\mathcal{H} = \{\psi(\vec{x})\}$

Rotate $\psi'(\vec{x}) = \psi(R^{-1}(g)\cdot\vec{x})$ ∞-dim rep

∞-dim rep separates into smaller reps

- Subspace of fixed energy
- Subspace of fixed angular momentum

Alemi: M&W next week!

- Tell us how to decompose group representations into irreducible pieces

Homework:

- Study degenerate ~~eigenvalues~~ energy eigenstates, different symmetries
- Decompose matrices into invariant pieces
- Write theories ~~for~~ ...