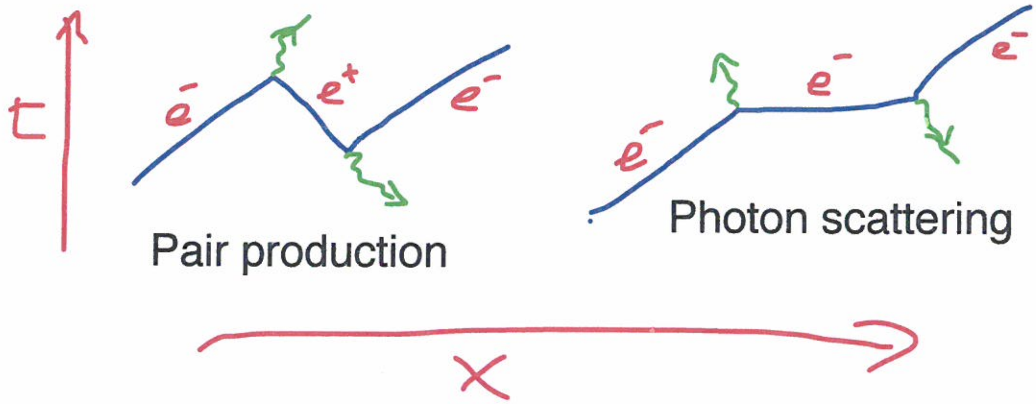
# Bosons and Fermions

Quantum identical particles are truly the same.




Pair production

Photon scattering

Wheeler:
Positron = same electron backward in time.

$$\Psi(\vec{r}_1,\vec{r}_2) = \underbrace{e^{i\varphi}}_{\pm 1}\,\Psi(\vec{r}_2,\vec{r}_1) = \underbrace{e^{2i\varphi}}_{1}\,\Psi(\vec{r}_1,\vec{r}_2)$$

(in 3D. In 2D more subtle: see Anyons, ps #4)

Bosons: +1
mesons, He4,
phonons, photons,
gluons, W+-, Z,
gravitons

Fermions: -1
electrons, protons,
neutrons, neutrinos,
quarks.

Spin statistics theorem
(relativistic QM):
integer spin = boson,
half-integer = Fermion

Q: Is a hydrogen atom a fermion or a boson?
Composites w/even # fermions = bosons

Q: Is Cesium? $I = 7/2$ nuclear spin
$S = 1/2$ electron spin

# Groups and Permutation Groups

A group G is a set {g1,...,g_N} closed under a product * that
is Associative: (g_i * g_j) * g_k = g_i * (g_j * g_k)
has an Identity e*g_i = g_i = g_i * e
has an Inverse for each element: g_i * (g_i^-1) = e

Q: Which are groups?
Integers Z under addition
Integers Z under multiplication
Rotations in SO(3)
SU(2) (spin 1/2 rotations)
Permutations of 5 particles

Q: How many elements are there in the permutation group of 5 particles?

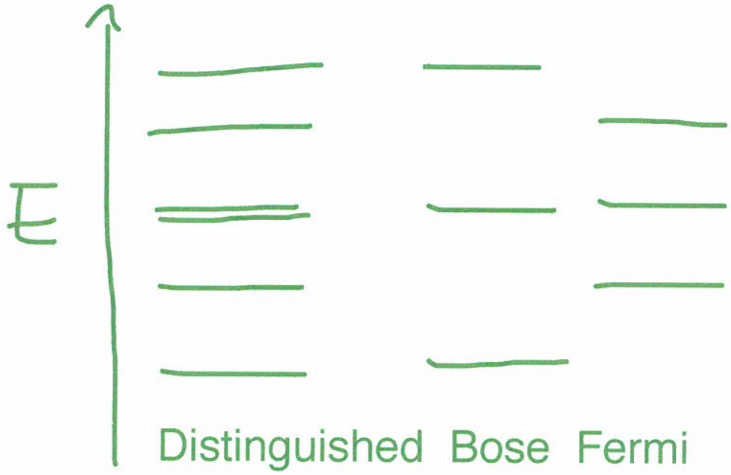
Q: In the permutation group of three particles
(e=123,132,213,231,312,321)
which are even and which are odd? Do the even permutations form a subgroup?

Many particles?
Permutation P = {P1,...,P_N} reordering integers {1,2,...,N}.
sigma(P)=+1 if P even permutation (net even number of swaps), sigma(P) = -1 for odd permutation

Bosons: $\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \Psi(\vec{r}_2, \vec{r}_1, \ldots, \vec{r}_N) = \Psi(\vec{r}_{P_1}, \vec{r}_{P_2}, \ldots, \vec{r}_{P_N})$

Fermions: $\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = -\Psi(\vec{r}_2, \vec{r}_1, \ldots, \vec{r}_N) = \sigma(P)\,\Psi(\vec{r}_{P_1}, \ldots, \vec{r}_{P_N})$

Fermion / bose wavefunctions confined to odd / even subspace of Hilbert space.




Q: Given eigenstates for a (distinguishable particle) Hamiltonian, how to form Bose & Fermi eigenstates?

A: Since [H,P] = 0, [H,sigma(P)]=0 too: eigenstates can be chosen to be either even or odd under permutations. Degenerate states with mixed symmetry can be symmetrized (Boson):

$$\Psi_{sym} = (\text{normalization}) \sum_P \Phi(\vec{r}_{P_1}, \ldots, \vec{r}_{P_N})$$

or antisymmetrized (Fermion):

$$\Psi_{asym} = (\text{norm}) \sum_P \sigma(P)\, \Phi(\vec{r}_{P_1} \ldots \vec{r}_{P_N})$$

If non-zero, some of energy-E eigenstates energy E.