Resonances

(AKA: Decaying 'eigenstates')

Remember 'triplet' state of He

$$\Psi^{*} = \underbrace{\psi^{*}(\vec{r}_1, \vec{r}_2)}_{\text{6 dim, antisymmetric}}\ \chi \quad \} \uparrow\uparrow \text{ e.g.}$$

Lifetime 7870 sec (Doubly forbidden)

Energy ~~−59.13~~ eV → Not an eigenstate?!

Complex energy $E_0 + i\Gamma = E$

$$e^{\frac{iEt}{\hbar}} = e^{iE_0t/\hbar}\, e^{i(i\Gamma)t/\hbar}$$

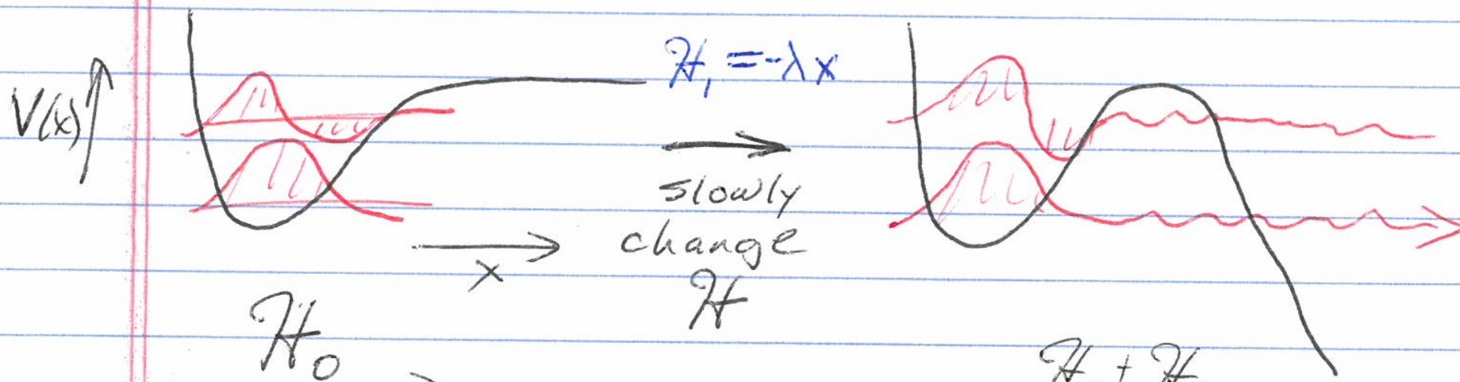
$$= e^{iE_0t/\hbar}\, e^{-t/\tau}$$

$\tau =$ ~~$1/\Gamma$~~ $\hbar/\Gamma = 7870$ sec, $\hbar = 6.58\times10^{-16}$ eVs

$$\boxed{\text{Energy} = -59.13 + i(8.36\times10^{-20})\ \text{eV}}$$

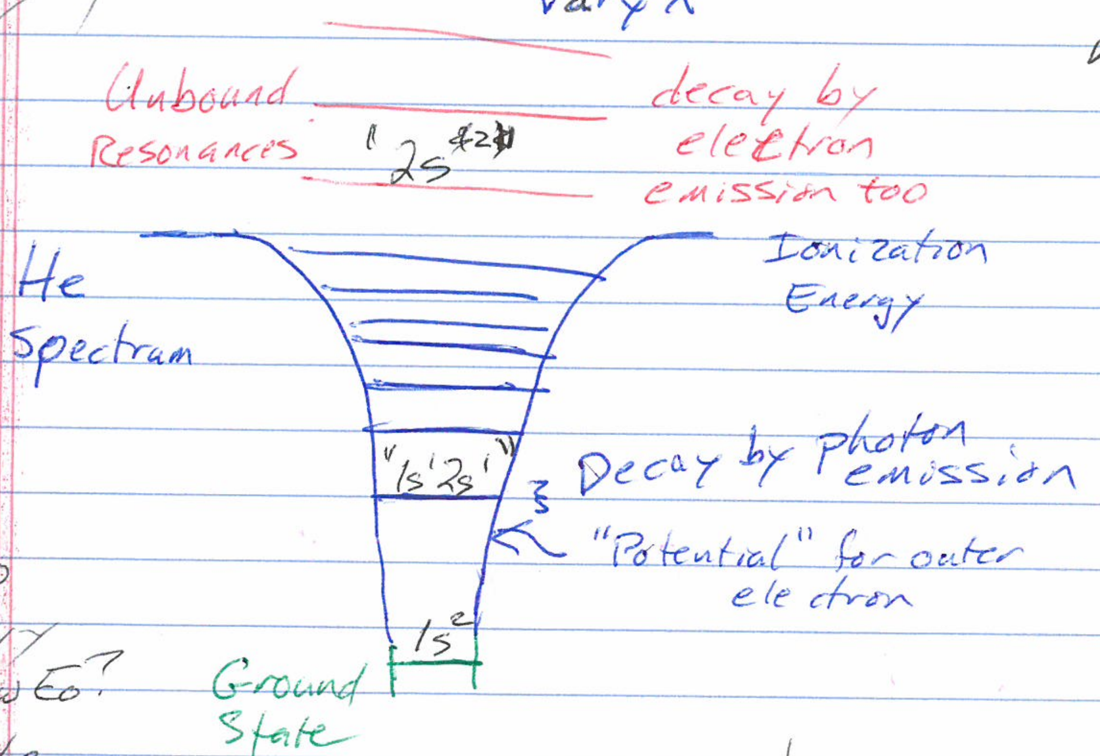
What the heck is a complex energy?

Resonances are 'former eigenstates' for simpler Hamiltonians

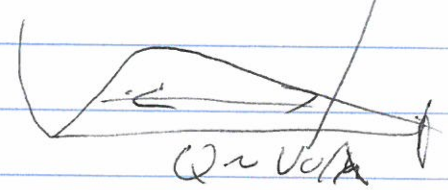


$\mathcal{H}_0 \longrightarrow \mathcal{H}_0 + \lambda \mathcal{H}_1 \longrightarrow \mathcal{H}_0 + \mathcal{H}_1$

Vary λ



Can we do pert theory to get new E_0? When will it be complex?



$$\mathrm{Im}(E) \propto e^{-\sqrt{2mV_0}\,Q/\hbar}$$

$$\propto e^{-\sqrt{2mV}\,V_0/\lambda\hbar} \sim e^{-c/\lambda}$$

} $\mathrm{Im}(E) = 0$ to all orders in pert theory

Q: What is Taylor series of $e^{-c/\lambda}$ A: 0

Heliam Hamiltonian:

$$\mathcal{H} = \frac{\left(P_1 - \frac{e}{c} A(r_1)\right)^2}{2m} + \frac{\left(\vec{P}_2 - \frac{e}{c}\vec{A}(r_2)\right)^2}{2m}$$

$$- \frac{Ze^2}{|\vec{r}_1|} - \frac{Ze^2}{|\vec{r}_2|} + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$$

$$+ \frac{1}{8\pi}\int \left(\vec{E}^2 + \vec{B}^2\right) d^3r$$

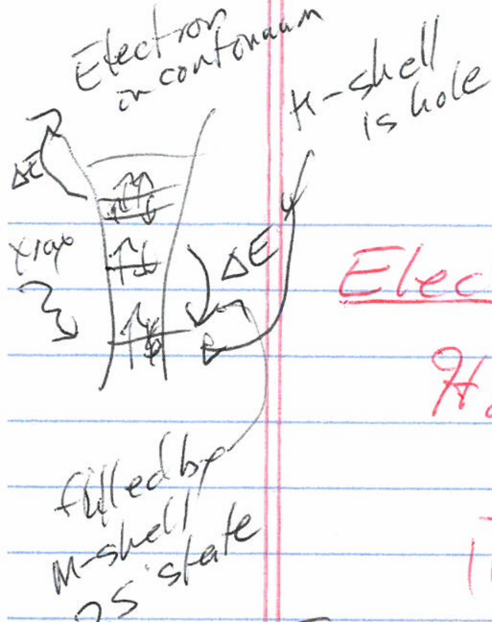
Q: What's Z? A: Two.

Photon Decay: $\mathcal{H}_0$ ignores coupling to electromagnetic field.

Ignore $e/c\, A$ terms.

Indeed, photon decay rates always have a factor of $\alpha = e^2/\hbar c = 1/137$.

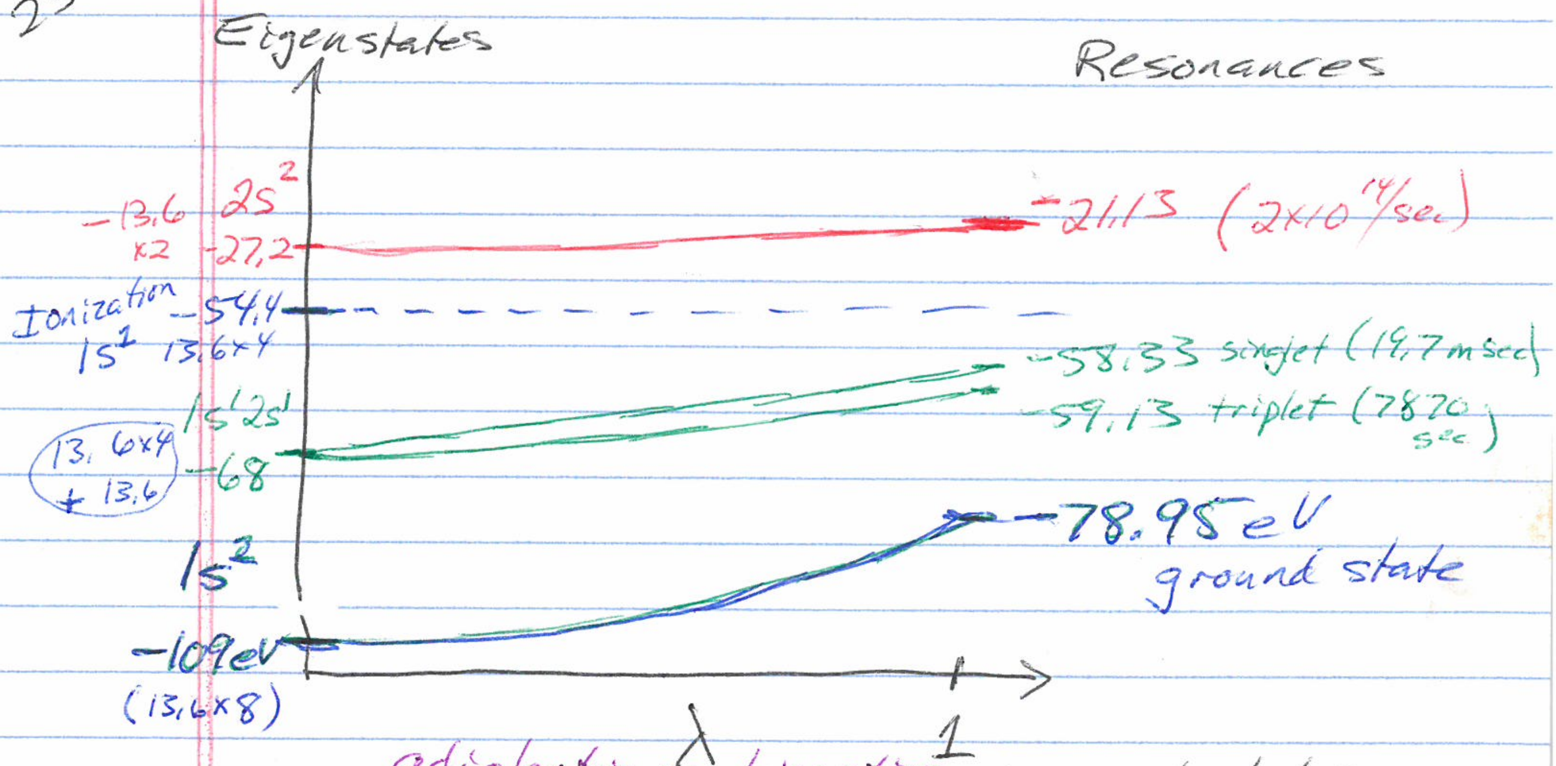
→ Perturbation theory 'valid' (except energy is complex.)



Electron emission (Auger process)

H_0 ignores photons <u>and</u> e-e repulsion

$\frac{\lambda e^2}{|r_1 - r_2|} \cdot \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$ <u>not small</u> (20% effect)



- We name the resonances and ground states by their 'unperturbed' eigenstates

Auger decay: $2s^2$ ⇒ $2s$ (Decay)

One electron $2s \to 1s$ — $1s$ ⇒ $1s^1$

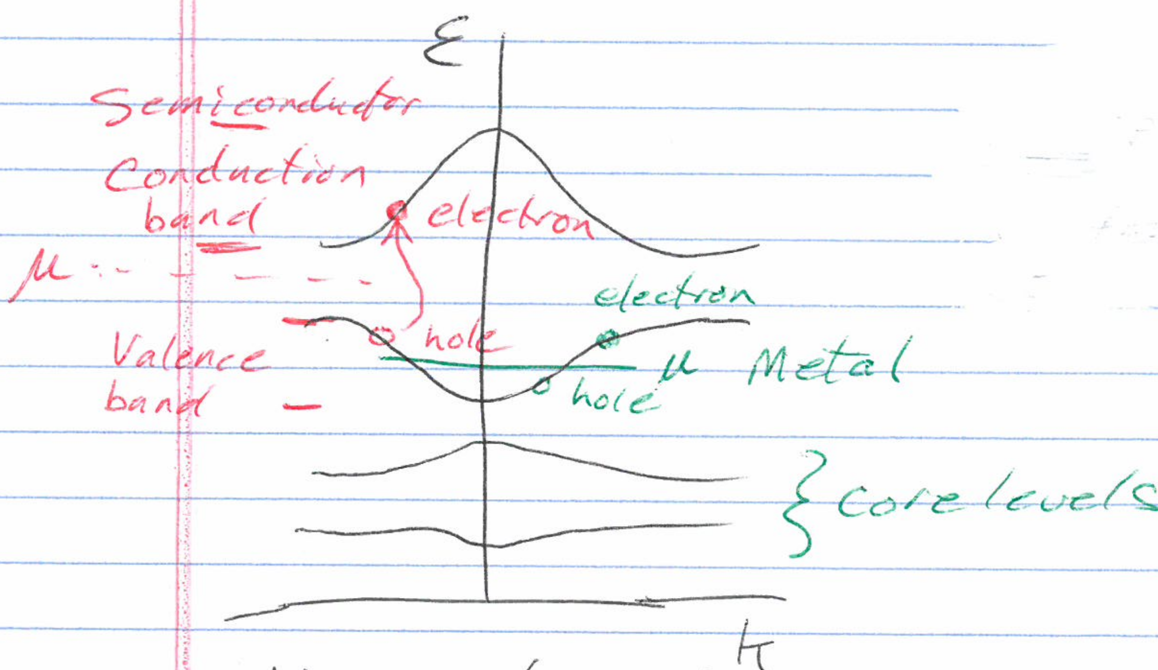
Other kicked out of atom

Periodic table: C $1s^2 2s^2 2p^2$ } Bogus, but useful

Resonances and Quasiparticles

Metals and Semiconductors:
Quasiparticles ~~comp~~ decay,
resonances labeled by non-interacting electron limit

- Atomic energy levels broaden into energy bands E(k) when put into crystal lattice



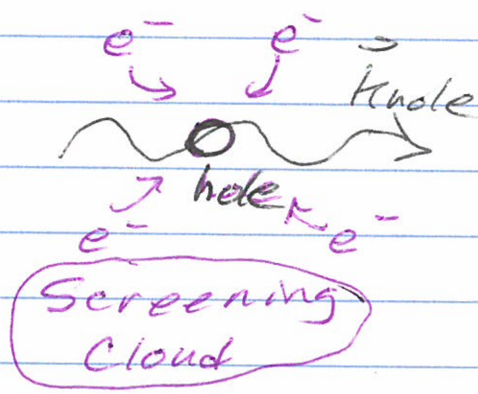
Non-interacting electron
single-particle eigenstates

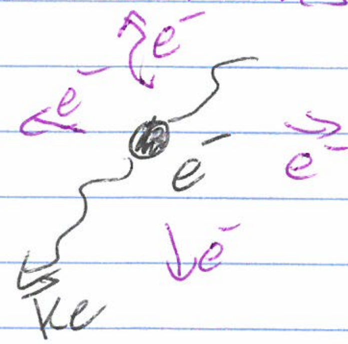
No e-e interactions: one e-h pair

$$\Psi_{k_h, k_e} = \left(\prod_{\substack{\varepsilon < \mu \\ k \neq k_{hole}}} \psi_k^{V}(r) \right) \psi_{k_{electron}}^{CB}(r)$$

Adiabatically turn on interactions

$\Psi^{*}_{k_h, k_e}$





Quasiparticle = Excitation + Screening Cloud

Quasiparticles <u>near</u> $\mu \cong \varepsilon_F$ have very long lifetimes