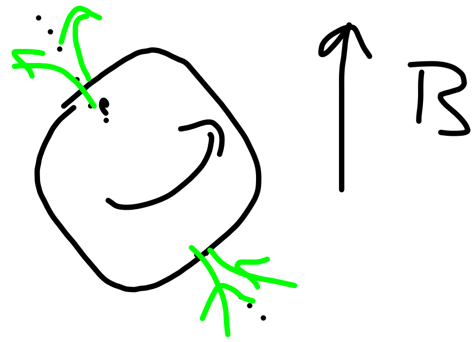
Review of Magnetic Moments

What happens to angular momentum in quantum mechanics?
Can we do an experiment to measure the angular momentum of an atom?

Currents feel magnetic fields.
Rotating atoms have currents.
How big a coupling?

Particle with spin in a magnetic field:
Classical spinning particle with charge density $\rho(\vec{r})$

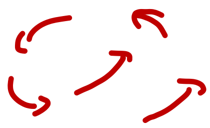


Magnetic Moment:

current loop

$\mu = IA/c = \frac{1}{2c} r(2\pi r I)$

current density

$\vec{J}(\vec{r})$  $\mu = \frac{1}{2c}\int \vec{r} \times \vec{J}\, dV$

energy of moment in field $-\vec{\mu}\cdot\vec{B}$

Note: Magnetic moment proportional to angular momentum $\vec{L}$

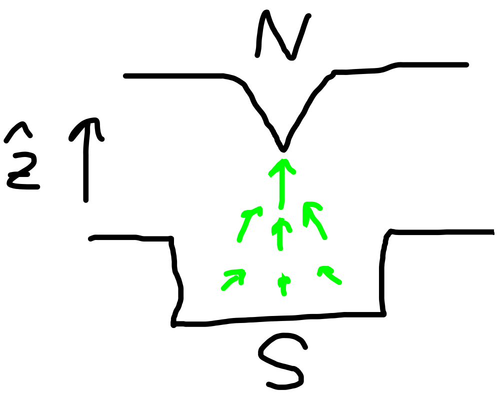
$$\vec{J} = \frac{e}{m}\vec{v}\,\rho(\vec{r})\,; \quad \vec{\mu} = \frac{1}{2c}\int \vec{r}\times\vec{J}\, d^3r$$
$$= \frac{e}{2mc}\int \rho(\vec{r})\, \vec{r}\times\vec{v}\, d^3r = \frac{e}{2mc}\vec{L}$$

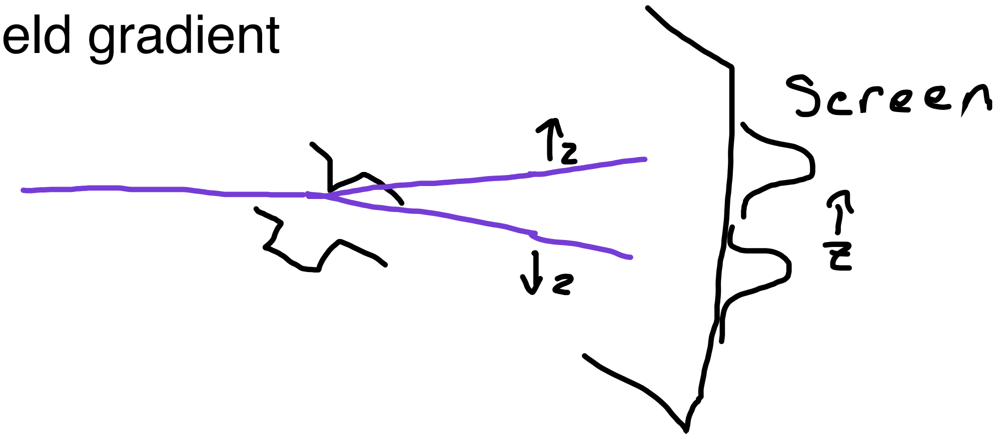
Note: heavier $\rightarrow$ smaller μ

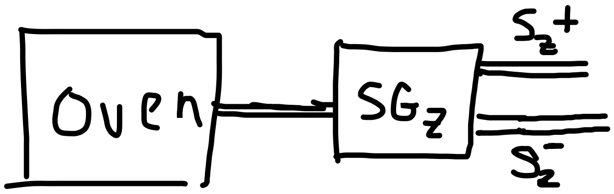
B twists angular momentum (hard to measure)
Gradient of B pulls spinning atom!

Stern Gerlach Experiment

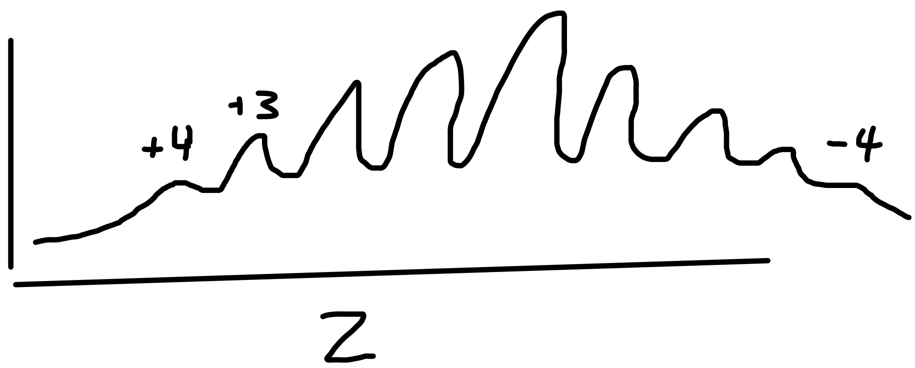
Pass beam through field gradient







Cesium Atoms



Magnetic moment quantized!

Rule: $L = m_z \hbar$, $m_z = \underbrace{-J, -J+1, \dots, J}_{2J+1 \text{ values}}$, $\boxed{\underbrace{J}_{J \text{ half integer}} = \text{Total Ang Mom}}$

Planck's constant
Quantum of angular momentum
$\hbar = 1.0546 \times 10^{-27}$ erg sec

Count the peaks. Angular momentum for Cs atom? $J = 4$

Where does it come from?

* Cs = [Xe] + 6s1; Xe core L=0, 6s1 L=0; electron S=1/2.
* Nucleus I = 7/2
* J = I x S = 7/2 x 1/2 = 4+3
* Hyperfine interactions select ground state J=4

Moment μ_z in experiment too big for Cs atom! $m = m_e$

$\mu_z = \underbrace{g}_{} \frac{e}{2mc} S_z$, $S_z = -4\hbar, -3\hbar, \ldots, 4\hbar$ Bohr Magneton $\frac{e\hbar}{2mc}$

Gyromagnetic Ratio

Gyromagnetic ratio depends on atom, nuclear spin, ...

$g_e \stackrel{\sim}{=} 2$ (Dirac equation, later)

$g_e = 2\left(1 + \frac{\alpha}{2\pi} + \ldots\right) = 2.00231930436 17(15)$ [QED]

- Best theory-experiment agreement in science
- Theory by Tom Kinoshita, Cornell prof (emeritus).

Ignore nucleus & nine beams. Pretend J=S=1/2, two beams

$S_z = \pm \hbar/2$

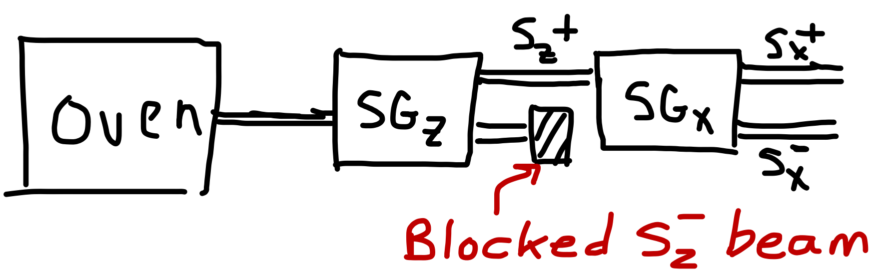
Operator

$$S_z = \begin{pmatrix} +\hbar/2 & 0 \\ 0 & -\hbar/2 \end{pmatrix} \qquad |\uparrow_z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ |\downarrow_z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \hbar/2\ \sigma_z \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ : Pauli Matrix}$$

Matrices, component vectors: Depend on basis
Operators, bras, kets: States independent of basis

What happens after a second SG filter, w/ rotated magnet?



$$S_x = \hbar/2\ \sigma_x = \hbar/2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ in } \sigma_z \text{ basis}$$

Eigenvalues $\pm \hbar/2$

Eigenstates $|\uparrow_x\rangle = \underbrace{\frac{1}{\sqrt{2}}}_{\text{Normalization}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle$

$|\langle\psi|\phi\rangle|^2 =$ Prob $|\phi\rangle$ is $|\psi\rangle$ so $|\langle\uparrow|\uparrow\rangle|^2 = 1$

$$|\downarrow_x\rangle = \frac{1}{\sqrt{2}}\underbrace{\begin{pmatrix}1\\-1\end{pmatrix}}_{\text{Orthogonal}} = \frac{1}{\sqrt{2}}|\uparrow_z\rangle - \frac{1}{\sqrt{2}}|\downarrow_z\rangle$$

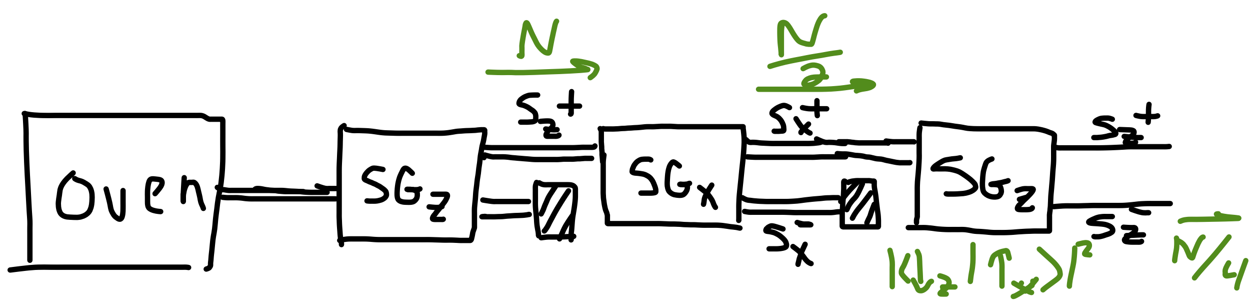
(Review of Hermitian operators?)

What fraction of the spins after the first filter have Sx = -1?

$\langle\downarrow_x|\underbrace{\uparrow_z\rangle}_{\text{First beam}} = 1/\sqrt{2}$, so $\underbrace{|\langle\downarrow_x|\uparrow_z\rangle|^2}_{\text{Prob } |\uparrow_z\rangle \text{ is } |\downarrow_x\rangle} = 1/2$

(Half, as one would expect from symmetry.)

What happens if I rotate and then rotate back?

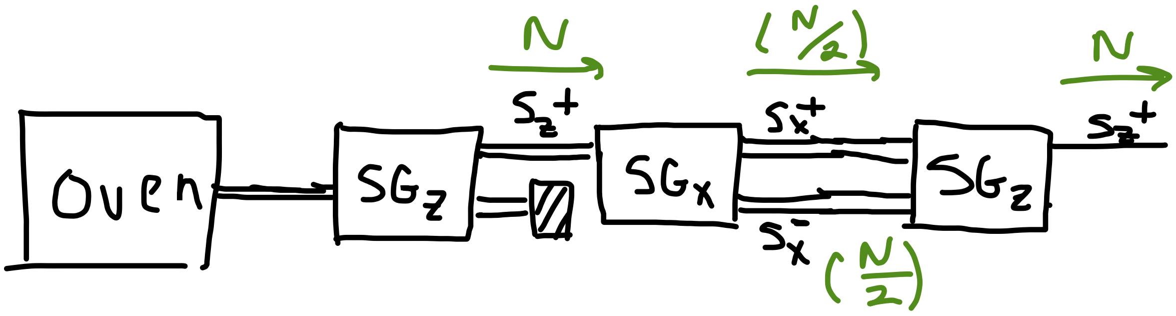


Half of the original sz+ particles removed by second block

Fraction of Sx+ beam now Sz- is $|\langle\downarrow_z|\uparrow_x\rangle|^2 = (1/2)$

Magnet SGx

But 1/4 of the particles now have Sz down! Not a "disturbance" due to the intervening magnet, because removing the block...



(and recombining the Sx beams) leaves 100% z-up spins.

So, magnet SGx doesn't erase the quantum information about sigma_z from the beam, the block does!

Close analogies to Feynman's discussion of double slit:

Review of Hermitian eigenvalues and eigenvectors...

Operator / Bra-ket notation:

ket $|\psi\rangle \xleftrightarrow{DC}$ Bra $\langle\psi|$ $\quad$ $O|\psi\rangle \xleftrightarrow{DC} \langle\psi|O^+$

Dual Correspondence $\quad$ Hermitian: $O^+ = O$

$$O|\lambda_i\rangle = \lambda_i|\lambda_i\rangle \xleftrightarrow{DC} \langle\lambda_i|\lambda_i^* = \langle\lambda_i|O^+$$

Phase of total wave function complex, but unmeasurable. Phase differences are physical. Observables measure real things -> Hermitian.

Matrix definition: $H = H^+ \equiv (H^T)^*$ Hermitian

(Complex vectors $u \cdot v = \langle u|v\rangle = (v \cdot u)^*$, so

$|u\rangle = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \Leftrightarrow \langle u| = (u_1^*, u_2^*, u_3^*)$ (Dual to $|u\rangle$ is $\langle u| = (u^T)^*$)

$$H|u\rangle = H\vec{u} \xleftrightarrow{DC} (H\vec{u})^{T*} = (u^T H^T)^* = \langle u|(H^T)^* = \langle u|H^+$$

Hermitian eigenvectors can be chosen to be orthonormal

$$\langle\lambda_i|\lambda_j\rangle = \langle\lambda_i|\left(O|\lambda_j\rangle\right)/\lambda_j = \left(\langle\lambda_i|O\right)|\lambda_j\rangle/\lambda_i$$

so unless the eigenvalues are equal, they better be orthogonal