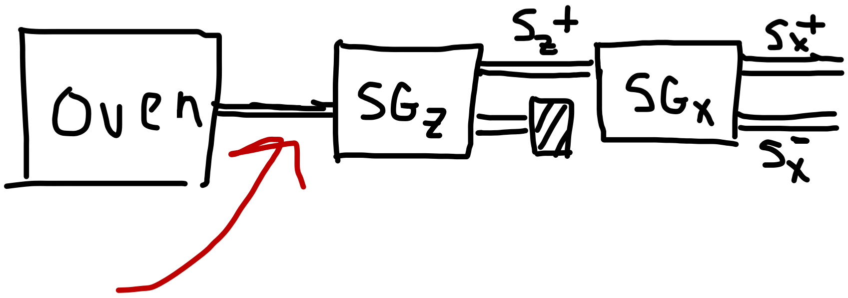
Entanglement



State of Stern Gerlach beam before first split? Unpolarized

Not $\underbrace{\frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle}_{\text{Quantum Superposition}}$! What is it? $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

Want Classical mixture "half" $|\uparrow_z\rangle$ & "half" $|\downarrow_z\rangle$

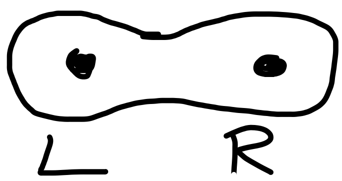
Leads us into entanglement (today) and density matrices (later)...

How is the atom beam created? Thought experiment.

Diatomic molecule w/ atoms w/ L=0, I=0, and S=1/2

(None exist: I checked.)

Ground state spin singlet.



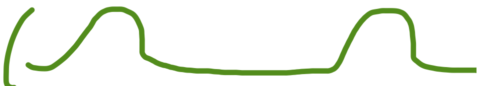
Spin state $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R\right)$

- Spin wavefunction (WF) antisymmetric: L -> R gives -1.
- Electrons are fermions.
- Net electron WF antisymmetric (nuclei = potential)
- Ignoring spin-orbit coupling (much later), WF factors

$$\Psi(x_R, s_R, x_L, s_L) = \chi(s_R, s_L)\,\psi(x_R, x_L) = \left(\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\right) \text{6 Dim}$$

so if $\chi(s_R, s_L) = -\chi(s_L, s_R)$, then $\psi(x_R, x_L) = \psi(x_L, x_R)$

Model oven splitting molecule, not altering spins

$\Psi = ($  $) \times \left(\left(|\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R \right) / \sqrt{2} \right.$

Two quantum systems are 'entangled' if

$|\psi_{AB}\rangle$ cannot be written as $|\psi_A\rangle |\psi_B\rangle$

Spin WFs of L & R are entangled. But how do we tell?

$|\uparrow_L\rangle |\downarrow\rangle_R$ entangled? No.

$\frac{1}{2}\left(|\uparrow\rangle_L |\uparrow_R\rangle + |\uparrow_L\rangle |\downarrow_R\rangle + |\downarrow\rangle_L |\uparrow\rangle_R + |\downarrow\rangle_L |\downarrow\rangle_R\right)$ entangled?

No: equals $|\uparrow_x\rangle_L |\uparrow_x\rangle_R = (|\uparrow_z\rangle_L + |\downarrow_z\rangle_L)(|\uparrow_z\rangle_R + |\downarrow_z\rangle_R)/2$

How do we tell?

Schmidt Decomposition = Singular Value Decomp (SVD)

Schmidt: $|\psi_{AB}\rangle = \sum_k \underbrace{\sigma_k}_{\text{positive}} \underbrace{|\varphi_A\rangle_k \, |\varphi_B\rangle_k}_{\text{orthonormal in } A, B}$

SVD: $\underbrace{M}_{\text{Any Matrix}} = U \underbrace{\Sigma}_{\substack{\text{Diagonal} \\ \sigma_{11} > \sigma_{22} > \dots >}} V^T$ } Columns of U, V orthonormal

$$M_{ij} = \sum_k U_{ik} \Sigma_{kk} V_{jk}$$

$$= \sum \sigma_{kk} \vec{u}^{(k)} \otimes \vec{V}^{(k)}$$

σ_{kk} unique: unless $\sigma_{11} = 1$, $\sigma_{22} = \dots = 0$, entangled

(Singular vectors almost unique, up to sign, degeneracy)

Is the singlet state entangled? Schmidt decomposition?

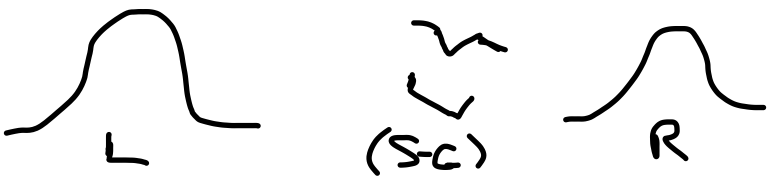
$\{|\uparrow\rangle_L, |\downarrow\rangle_L\}$ orthonormal in L

$\{-|\uparrow\rangle_R, |\downarrow\rangle_R\}$ orthonormal in R

$$\text{Singlet} = \frac{|\uparrow\rangle_L|\downarrow\rangle_R - |\downarrow\rangle_L|\uparrow\rangle_R}{\sqrt{2}} = \underbrace{\frac{1}{\sqrt{2}}}_{\sigma_{11}} |\uparrow_L\rangle|\downarrow\rangle_R + \underbrace{\frac{1}{\sqrt{2}}}_{\sigma_{22}} |\downarrow\rangle_L(-|\uparrow\rangle_R)$$

Since SVD / Schmidt decomposition unique, singlet entangled

Einstein: Spooky action at a distance



What happens when S-G measures spin SzR of R atom?

$$\chi = \left(\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}\right) = \left(\frac{\uparrow_L}{\sqrt{2}}\right)\downarrow_R + \left(\frac{\downarrow_L}{\sqrt{2}}\right)\uparrow_R \Rightarrow \begin{cases} \downarrow_L \uparrow_R & \text{prob } \left(-\frac{1}{\sqrt{2}}\right)^2 \\ \uparrow_L \downarrow_R & \text{prob } \left(\frac{1}{\sqrt{2}}\right)^2 \end{cases}$$

Instantly, left atom has definite S_z^L!

Copenhagen: Wavefunction 'collapse' (above)

Many worlds: Universe splits

$$\frac{1}{\sqrt{2}} |\uparrow_L \downarrow_R\rangle |\text{Screen down dot}\rangle - \frac{1}{\sqrt{2}} |\downarrow_L \uparrow_R\rangle |\text{Screen up dot}\rangle$$

Mermin 'Ithaca' interpretation: Quantum theory tells correlations: If R measures up, then L will measure down, and vice-versa

S-G Beam is R's leaving L's behind!
R-beam forgetting L 'half' up and 'half' down (density matrix, later)
Entropy = Quantification of amount of ignorance (general)

$$S = -k_B \sum P_i \log P_i \quad \text{for } P_i = \begin{Bmatrix} \text{classical} \\ \text{probability} \\ \text{state } i \end{Bmatrix} = \sigma_{ii}^2$$

Entanglement entropy = Entropy of R if L is forgotten

$$S = -k_B \sum_i \sigma_i^2 \log \sigma_i^2 = -\frac{k_B}{2} \log \tfrac{1}{2} - \frac{k_B}{2} \log \tfrac{1}{2}$$

$$= k_B \log 2 = k_B \log(\text{\# of equally likely states})$$

Ignorance

Note: $\sigma_{ii} \to \sigma_i$ (Convention)
$\log \equiv \log_e = \ln$ (10 isn't important to physics)
k_B = Boltzmann's constant
sometimes $\to k_B = 1/\log 2$ so
entropy measured in bits
Sometimes $k=1$, entropy in 'nats'

Entanglement entropy quantifies information lost when state of L is forgotten (coherence disrupted)...