Aharonov-Bohm Effect

Consider 'covariant derivative' of vector $v$ tangent to curved surface.
Parallel transport of $v$ around closed loop rotates vector

$$D_\alpha v^\beta = (\partial_\alpha v^\beta + \Gamma^\beta_{\alpha\gamma} v^\gamma) = 0$$

*Covariant Derivative*

* Falling Cat: Closed loop in shape space leads to rotation
* Swimming Bacteria (low Re): Closed loop in shape space leads to translation
* Anyon: Transporting excitation of 2D electron gas in field (FQHE) around loop in space leads to fractional statistics
* Blue Phase Frustration (HW): Following local low energy structure around loop leads to mismatch -> defect lattice formation

Today: What is the parallel transport of a phase around a closed loop $C$?

$$D_\alpha = \partial_\alpha - \frac{i q}{\hbar c} A_\alpha$$

$$(\partial_\alpha + i \frac{q}{\hbar c} A_\alpha) e^{i \phi(x)} = (\nabla_\alpha \phi + i \frac{q}{\hbar c} \vec{A}) e^{i \phi} = 0$$

$$\oint_C \nabla \phi \cdot dl = \frac{q}{\hbar c} \oint_C A \cdot dl$$

$$= \frac{q}{\hbar c} \Phi_B$$  \text{flux enclosed}$$

Consider impenetrable solenoid, no B field outside

If $\Phi_B = n \frac{\hbar c}{2} = 2\pi n \frac{\hbar c}{2}$\n
$\Delta \Phi = 2\pi n$, no effect.\n
No surprise? $B=0$\n
If $\Phi_B \neq n \frac{\hbar c}{e}$, A field affects WFs outside, even though $B=0$!
Dirac Magnetic Monopole

If a magnetic monopole exists anywhere, it forces quantization of electric charge too.

\[ B \propto \hat{r} \text{ minimizes energy} \]

\[ \oint B \cdot d\mathbf{S} = \Phi_0 \]

\[ \mathbf{B} = \frac{\hbar c}{e} \frac{\hat{r}}{4\pi r^2} = \frac{\hbar c}{2e} \frac{\hat{r}}{r^2} = 2m \frac{\hat{r}}{r^2} \]

just like Coulomb law \( \mathbf{E} = \frac{e \hat{r}}{r^2} \),

Magnetic charge must satisfy \( e_2 m = \frac{\hbar c}{2} n, \quad 2m = \frac{\hbar c}{2e} \frac{n}{2\alpha} \approx \frac{137}{2} n \quad \Rightarrow \quad e \frac{\hbar c}{n} = \alpha \approx \frac{1}{137} \)

If a magnetic monopole exists anywhere, it forces quantization of electric charge too.