Aharonov-Bohm Effect

Consider 'covariant derivative' of vector v tangent to curved surface
Parallel transport of v around closed loop rotates vector

$$D_\alpha v^\beta = \underbrace{\left(\partial_\alpha v^\beta + \Gamma^\beta_{\alpha\gamma} v^\gamma\right)}_{\text{Covariant Derivative}} = 0$$

* Falling Cat: Closed loop in shape space leads to rotation
* Swimming Bacteria (low Re): Closed loop in shape space leads to translation
* Anyon: Transporting excitation of 2D electron gas in field (FQHE) around loop in space leads to fractional statistics
* Blue Phase Frustration (HW): Following local low energy structure around loop leads to mismatch -> defect lattice formation

Today: What is the parallel transport of a phase around a closed loop C? $D_\alpha = \partial_\alpha - \frac{iq}{\hbar c} A_\alpha$

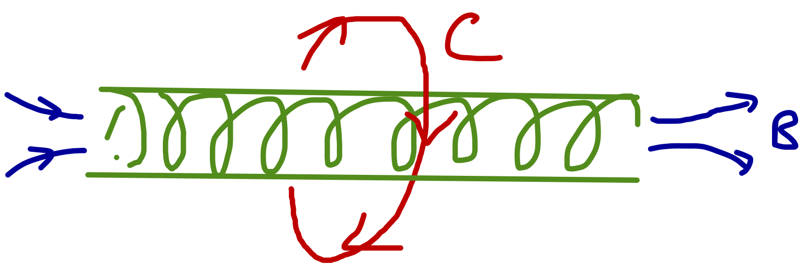
$$\left(\partial_\alpha + i\frac{q}{\hbar c} A_\alpha\right) e^{i\varphi(\vec{x})} = \left(\cancel{i}\nabla_\alpha \varphi + \cancel{i}\frac{q}{\hbar c}\vec{A}\right) e^{i\varphi} = 0$$

$$\int_C \nabla\varphi \cdot d\ell = \frac{q}{\hbar c}\int_C A \cdot d\ell$$

$$= \frac{q}{\hbar c}\Phi_B \quad \text{flux enclosed}$$

$B = \text{curl} A$
so $\int A \cdot d\ell = \int \text{curl} B \cdot d\ell$
$= \int B \cdot ds = \Phi_B$

Consider impenetrable solenoid, no B field outside

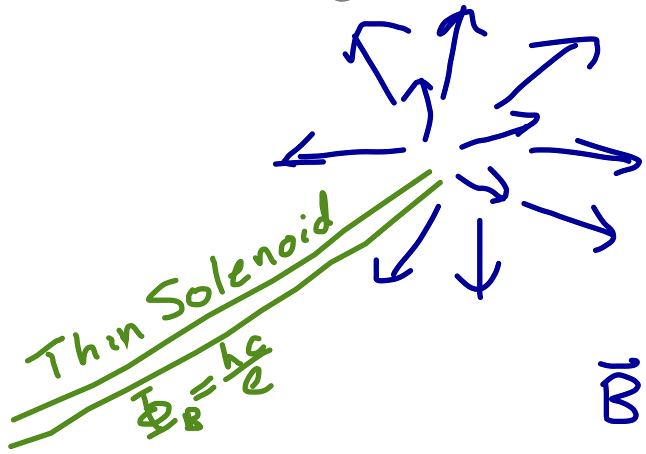


If $\Phi_B = n\frac{hc}{q} = 2\pi n\frac{\hbar c}{q}$
$\Delta\varphi = 2\pi n$, <u>no effect</u>.

No surprise? $B = 0$

If $\Phi_B \neq n\Phi_0 = n\frac{hc}{e}$, A field affects WFs outside, even though B=0!

Dirac Magnetic Monopole



$B \propto \hat{r}$ minimizes energy

$\int \vec{B} \cdot d\vec{S} = \Phi_0$

$$\vec{B} = \frac{hc}{e}\frac{\hat{r}}{4\pi r^2} = \frac{\hbar c}{2e}\frac{\hat{r}}{r^2} = q_m \frac{\hat{r}}{r^2}$$

just like Coulomb law $\vec{E} = \frac{e\hat{r}}{r^2}$,

Magnetic charge must satisfy $e q_m = \frac{\hbar c}{2} n$, $\frac{q_m}{e} = \underbrace{\frac{\hbar c n}{2e^2}}_{e^2/\hbar c = \alpha \sim \frac{1}{137}} = \frac{n}{2\alpha} \cong \frac{137}{2} n$

If a magnetic monopole exists anywhere, it forces quantization of electric charge too.