Disorder-driven first-order phase transformations: A model for hysteresis

Karin Dahmen, Sivan Kartha, James A. Krumhansl,[a] Bruce W. Roberts,
James P. Sethna, and Joel D. Shore[b]
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

Hysteresis loops in some magnetic systems are composed of small avalanches (manifesting themselves as Barkhausen pulses). Hysteresis loops in other first-order phase transitions (including some magnetic systems) often occur via one large avalanche. The transition between these two limiting cases is studied, by varying the disorder in the zero-temperature random-field Ising model. Sweeping the external field through zero at weak disorder, we get one large avalanche with small precursors and aftershocks. At strong disorder, we get a distribution of small avalanches (small Barkhausen effect). At the critical value of disorder where a macroscopic jump in the magnetization first occurs, universal power-law behavior of the magnetization and of the distribution of (Barkhausen) avalanches is found. This transition is studied by mean-field theory, perturbative expansions, and numerical simulation in three dimensions.

The shapes of hysteresis curves in magnetic materials vary widely for different materials. This variation manifests itself in variation of parameters, such as the coercivity, remanence, and susceptibility. Of particular interest to us is the variation of the sizes of Barkhausen noise pulses.[1] These pulses are a manifestation of avalanches taking place in the magnetic system, with groups of domains (or spins) flipping over in one avalanche. One distinguishes between the small Barkhausen effect and large Barkhausen discontinuities. The small Barkhausen effect is the observation of small pulses representing small avalanches in the system. None of these pulses sweep through the entire system. Often, soft magnetic materials after annealing show mainly the small Barkhausen effect.[2] There are other materials that have hysteresis loops that are more square and have large jumps in the magnetization. These jumps are often called "large Barkhausen discontinuities"[2] in magnetic materials and "bursts" in a similar effect in Martensites.[3] These large jumps represent avalanches that sweep through a sizable fraction of the system. In the thermodynamic limit, we call avalanches that sweep a finite fraction of the system "infinite avalanches." There exist models like the Stoner–Wohlfarth model and the Preisach model,[2,4] that can describe both types of hysteresis loops. The Preisach model, and other models, break up the hysteresis loop into contributions from elementary hysteresis domains, each with their own value for remanence and coercivity, with no interactions between the domains.

We study hysteresis in the zero temperature random-field Ising model (RFIM), where *interaction* between hysteresis domains (spins) is implicit.[5] We note that we are not considering the equilibrium RFIM or any $T\rightarrow 0$ limit of that model. The dynamics of the approach to thermal equilibrium in that model has been well studied.[6] However, we are interested in a model with the same Hamiltonian as the equilibrium RFIM, but with a (very natural but nonequilibrium) specified dynamics.[5] By varying the randomness, we can use this RFIM to describe hysteresis loops, both with and without large Barkhausen discontinuities. The most interesting consequence, however, of introducing interactions between the magnetic domains, is that the transition from infinite avalanche to noninfinite avalanche hysteresis loops is accompanied by critical fluctuations and diverging correlation lengths. As one knows from the theory of phase transitions,[7] this allows one to write down scaling laws with universal exponents that are valid near the transition point. By "universal," we mean that the behavior is independent of many details of the system (and thus the same for theoretical models and real materials), and depends only on the dimension, the range of interaction, and the symmetries of the order parameter.

We consider a simple cubic lattice. Each site i in this lattice can represent an entire magnetic domain. The magnetization at site i is s_i. It can be pointing either up or down, i.e., $s_i=\pm 1$. The Hamiltonian of the RFIM is

$$\mathscr{H}=-\sum_{\langle ij\rangle} J s_i s_j-\sum_i (f_i s_i+H s_i), \tag{1}$$

where the first sum is over nearest-neighbor pairs. This term represents a ferromagnetic interaction of magnitude J between the domains. At each site in the crystal a random magnetic field f_i is added to model disorder (like defects, etc.) in the crystal. We assume a Gaussian distribution of width R for these random fields. H is the strength of the external, homogeneous magnetic field. As we change the external magnetic field H adiabatically, the system does not relax to its true equilibrium ground state. Instead, it stumbles from one metastable state to another in response to the changing field. For experimental systems with high enough barriers to relaxation for each elementary domain, such that thermal fluctuations can be ignored on experimental time scales, this assumption that the system does not relax to its true ground state is valid. This applies to most magnetic memory devices (by design), and also other systems, like athermal Martensites.[3]

A domain on a magnetic tape will flip over when the force from the external field exceeds the coupling forces due to the neighbors plus any forces from random anisotropies or random fields within the domain. Our model uses precisely this dynamics. To define a dynamics for this system, we as-

[a]Present address: 515 Station Road, Amherst, Massachusetts 01002.
[b]Present address: Department of Physics, Simon Fraser University, Burnaby, British Columbia, V5A 1S6, Canada.

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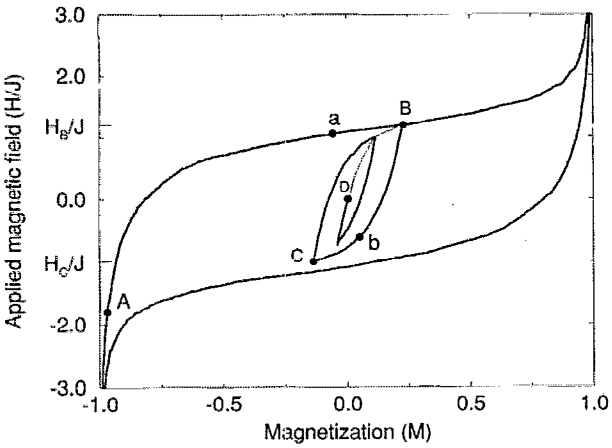

FIG. 1. Hysteresis loop showing return-point memory. The magnetization as a function of external field for a 30^3 system with disorder $R=3.5$ J. If the field H is made to backtrack from H_B to H_C, and swept up to H_B again, it returns to precisely the same state at H_B, from which it left the outer loop. It remembers this "return-point," even if it is forced to run through subcycles within cycles, and so on, as the figure shows. A simple proof for this "return-point memory" in certain systems is given elsewhere.[5]

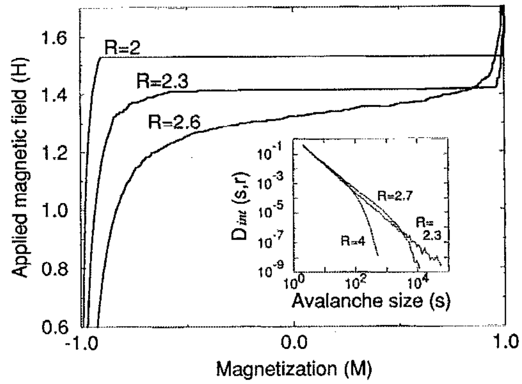

FIG. 2. Varying the disorder. Three $H(M)$ curves for different levels of disorder, for a 60^3 system. Our current estimate of the critical disorder is $R_c=2.23$ J (we set $J=1$). At $R=2<R_c$, there is an infinite avalanche that seems to appear abruptly. For $R=2.6>R_c$ the dynamics is macroscopically smooth, although, of course, microscopically it is a sequence of sizeable avalanches. At $R=2.3$, near the critical level of disorder, extremely large events become common. Inset: Log–log plot of the integrated avalanche-size distribution $D_{\rm int}(s,r)$ vs avalanche size s. $D_{\rm int}(s,r)$ is the integral of $D(s,r,h)$ (see the text) over one sweep of the magnetic field from $-\infty$ to $+\infty$, averaged here over five systems of size 120^3 and plotted at $R=2.3$, $R=2.7$, and $R=4.0$. Notice the power-law region $D_{\rm int}(s)\sim s^{-(\tau+\sigma\beta\delta)}$ and the cutoff at $s_{\rm max}\sim(R-R_c)^{-1/\sigma}$.

sume that as the external magnetic field is changed adiabatically, each domain will flip when the direction of its total local field,

$$F_i=\sum_j Js_j+f_i+H, \tag{2}$$

changes. Initially, all domains are pointing down at a sufficiently negative H. The system then transforms from negative to positive magnetization as the field is swept upward. Related approaches were discussed in Ref. 5. In addition, other related work has come to our attention.[8] The result of a simulation of this system for a width of the Gaussian distribution of random fields $R=3.5$ J in three dimensions is shown in Fig. 1. The outer loop shows the external magnetic field being slowly increased until all domains are pointing up and then decreased back to the initial value. Figure 2 shows the upper branch of three similar hysteresis loops at different disorders R. One sees that for increasing values of R the size of the jump or large Barkhausen effect goes to zero and the hysteresis loops look softer. In fact, one expects that for small R compared to the coupling J, one of the first domains to flip will push over its neighbors, which will cause their neighbors to flip and so on, until a finite fraction of the system is transformed in one "infinite avalanche," which manifests itself as a jump in the magnetization curve. The critical magnetic field $H_c^u(R)$ at which this jump occurs in the upper branch of the hysteresis loop, decreases monotonically with increasing disorder R, starting from $H_c^u(0)=zJ$, where z is the coordination number of the crystal. Experiments,[3] as well as our numerical simulation[5] and analytical calculations[9] suggest that the onset of the infinite avalanche at $H_c^u(R)$ seems to be abrupt for $R<R_c$ in three dimensions, as H is slowly changed.

In the other regime, where the disorder R is large compared to the coupling J, each domain will essentially flip on its own. There will be only small avalanches happening. Empirically, the coercivity decreases further for increasing R. There must be a critical disorder R_c separating these two regimes. In other words, the line of critical fields $H_c^u(R)$ at which an infinite avalanche occurs, ends at the point $H_c^u(R_c)$. For $R>R_c$ the magnetization curves do not show any infinite avalanches. This has been verified both experimentally[3] and in our simulation in three dimensions ($R_c=2.23$ J numerically). In two dimensions, the numerical results suggest that R_c is zero. We have found numerical[5] and analytical[5,9] evidence that the transition occurring at R_c from infinite avalanche to noninfinite avalanche hysteresis loops is continuous: One finds avalanches of all sizes at that point. As one approaches the critical end point at $[R_c,H_c^u(R_c)]$ in the (R,H) plane, we find diverging correlation lengths and presumably universal critical behavior. This transition between the two regimes has a natural parallel with ordinary equilibrium temperature-driven phase transitions. To make this parallel concrete, consider the pure equilibrium Ising model in three dimensions. In this model there is a transition at a certain temperature where the order parameter (which in this case is the magnetization) takes on a finite value. This transition occurs when the ordering tendency of the bonds between spins overcomes the disordering tendency of thermal fluctuations. In our model, the order parameter is the size of the infinite avalanche $\Delta M[H_c(R)]$. We have a finite value for our order parameter when the "ordering" tendency of the bonds between spins overcomes the disordering tendency of the random fields. For us, ordering is represented by the existence of an infinite avalanche, where a large number of domains flip at one field. We also note that the analog of the equilibrium correlation length for our model is the size of the largest finite avalanche.

From both mean-field approaches and renormalization

TABLE I. Universal exponents for critical behavior in hysteresis loops. The exponents β and δ tell how the magnetization scales with $r=(R_c-R)/R_c$ and $h=H-H_c$, respectively, for example, $\{M(H)-M[H_c^u(R_c)]\} \sim (H-H_c)^{(1/\delta)}$ at R_c. The correlation length exponent ν is measured (numerically) using finite-size scaling. The exponent σ tells how the cutoff of the avalanche size distribution scales with r, and τ is the exponent of the power-law decay of the differential avalanche size distribution $D(s,r,h)$ at the critical point ($r=0$ and $h=0$).

Exponent	ϵ expansion with $\epsilon=6-d$, at $\epsilon=3$	Simulation in three dimensions
$1/\nu$	$2-\epsilon/3=1$	1.0 ± 0.1
β	$0.5-\epsilon/6=0$	0.17 ± 0.07
$\beta\delta$	$1.5+O(\epsilon^2)=1.5$	2.0 ± 0.3
δ	$3+\epsilon=6$	(around 12)
$1/\sigma$	$\cdots$	2.9 ± 0.15
τ	$\cdots$	1.35 ± 0.2

group techniques, near R_c we expect universal scaling laws for the behavior of the magnetization M and the distribution D of sizes s of avalanches occurring upon increasing H by a differential amount dH. For $r\equiv(R_c-R)/R_c$ and $h\equiv[H-H_c^u(R_c)]$, we have $M(h,r)\sim|r|^\beta \mathscr{M}_\pm(h/|r|^{\beta\delta})$, and $D(s,r,h)\sim s^{-\tau}\mathscr{D}_\pm(s/|r|^{-1/\sigma},h/|r|^{\beta\delta})$, where $\pm$ refers to the sign of r and $\mathscr{M}_\pm$ and $\mathscr{D}_\pm$ are universal scaling functions. In mean-field theory, where we couple every domain with all N other domains with coupling J/N, we find $\delta=3$ and $\beta=\frac{1}{2}$. Near the end point, the jump in the magnetization ΔM due to the infinite avalanche scales like r^β. Furthermore, in mean-field theory, $\tau=\frac{3}{2}$ and $\sigma=\frac{1}{2}$. We expect these exponents to be accurate in six and all higher dimensions. We have performed an expansion around six dimensions to obtain an analytical prediction for these exponents in three dimensions, using renormalization group techniques.[9] Independently, we have determined the exponents from the numerical simulation in three dimensions.[5] Table I gives the results from both approaches.

Particularly interesting for comparison with experiments[1] is the avalanche size distribution integrated over a whole hysteresis loop (see the inset of Fig. 2). In fact, the distribution of avalanches, as it is measured in the Barkhausen effect, has been studied, and some preliminary fits to power laws were made.[1] These fits have been interpreted as an indication of self-organized criticality. The noticeable cutoff of the power law avalanche size distribution was thought to be due to finite size effects. In the view of our model, we would suggest that the power laws might be an indication of an ordinary critical point instead of self-organized criticality. We would expect the cutoff in the distribution to move to larger and larger avalanche sizes, as the critical point is approached by tuning the randomness R to its critical value R_c. We think that similar effects might be observed, for example, upon changing the distribution of random anisotropies. (Though the fact that random magnetic fields break time-reversal symmetry, whereas random anisotropies do not, could change the values of the critical exponents.) Furthermore, we expect this effect not only to be found in magnetic materials. Indeed, for a FeNi alloy, which is an athermal Martensite, an increase in grain sizes has been seen to lead to a crossover from noninfinite avalanche to infinite avalanche behavior.[3]

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